Instituto Superior Técnico
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Mathematical Modeling Issues in Future Wireless Networks

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Outline

- Research and Development of Emerging 5G Technologies for Digital Economy
- Ongoing Projects
    - Resource allocation in wireless networks with random resource requirements
    - Resource allocation in wireless networks for Licensed Shared Access (LSA)
    - Stochastic geometry models and SIR analysis in D2D wireless networks
    - Modelling users' mobility
- Future Projects

Collaboration agreements and partner universities

Tampere University of Technology, Finland

Brno University of Technology, Czech Republic

University Mediterranea of Reggio Calabria, Italy

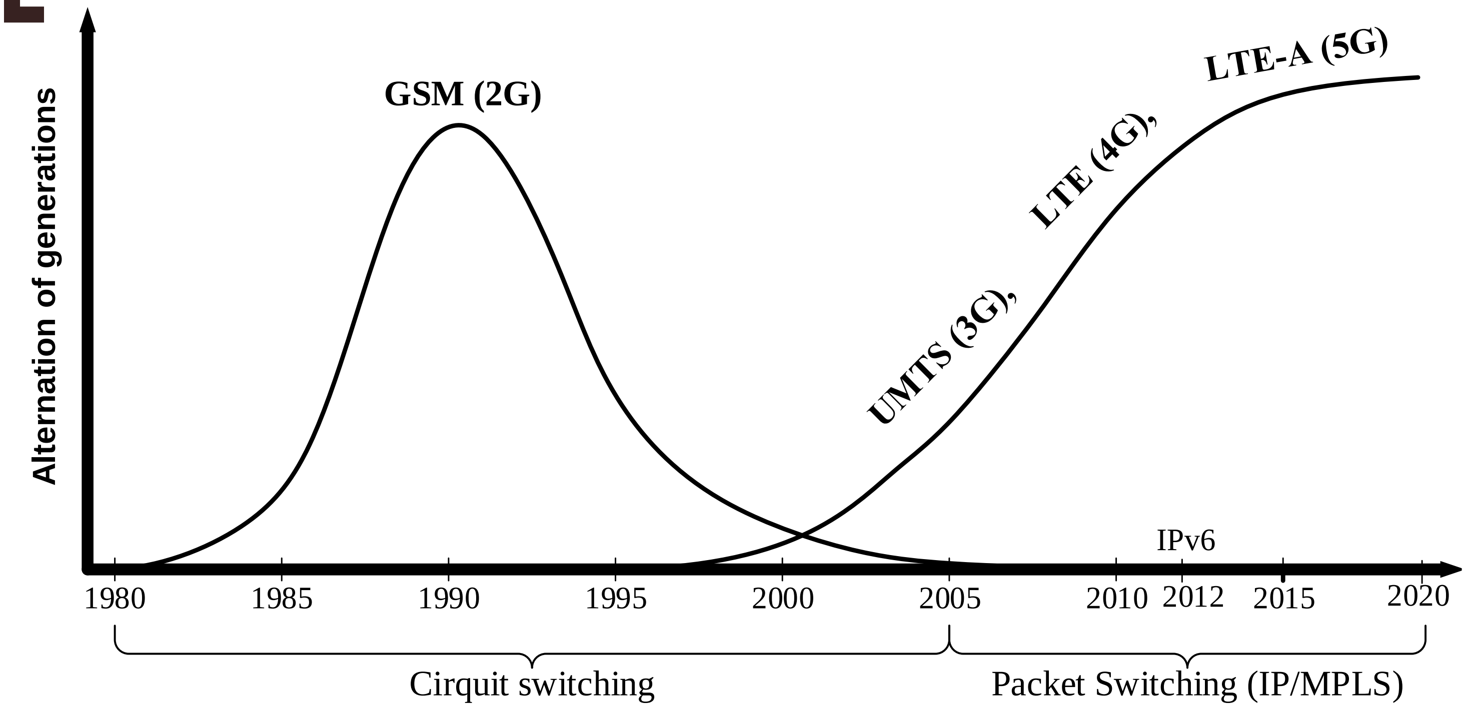
University of Pisa, Italy

Bonch-Bruevich Saint - Petersburg State University of Telecommunications, Russia

Keldysh Institute of Applied Mathematics (Russian Academy of Sciences), Russia

Federal Research Center "Computer Science and Control" (Russian Academy of Sciences), Russia

New paradigm in mobile communications

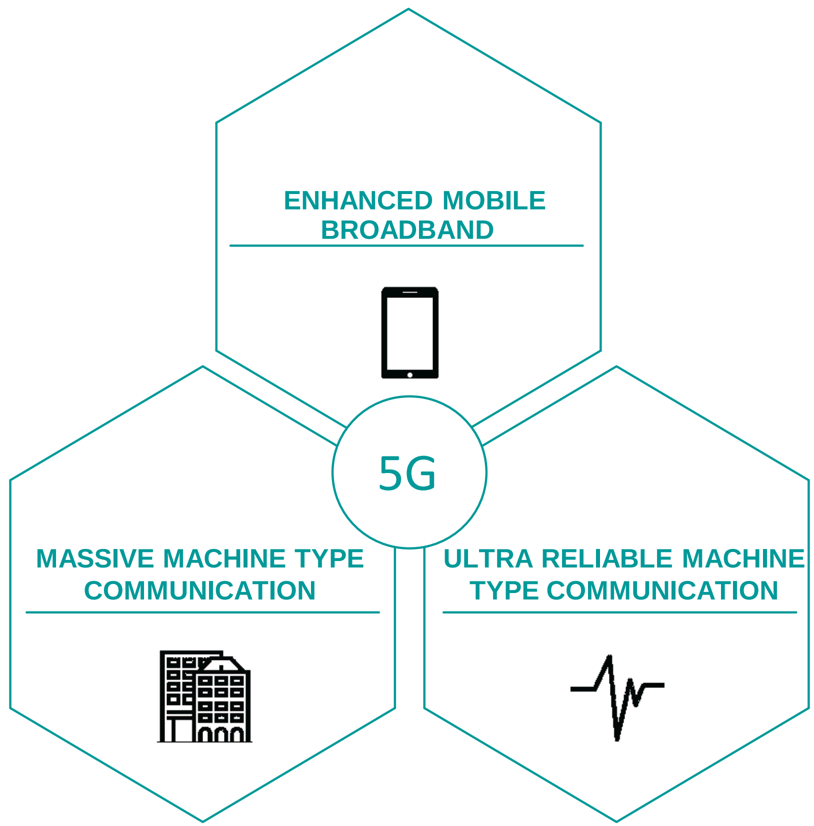


Currently, there is a fundamental change, a shift in the paradigm of the telecommunications infrastructure, a shift that in its significance significantly exceeds the changes in personal communications caused by the transition from the telegraph to the telephone. This shift is the transition from circuit-switched (CS) networks to packet-switched networks (IPs) based on IP technologies.

(The paradigm the initial conceptual model of statement of problems and their solutions, as well as research methods prevailing during a certain historical period in the scientific community)

ITU Vision for IMT-2020 and Beyond

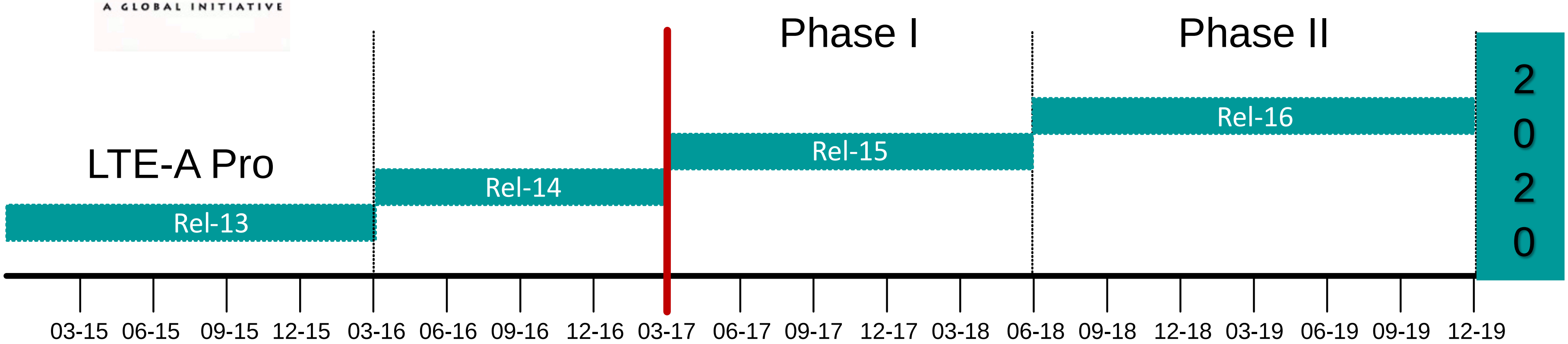
> 10 Gbps peak rates
100 Mbps average rates



> 1M / km^2 Connections
for
Smart Society

< 1 ms Latency
for
Tactile Internet,
autonomous
transport systems,
augmented reality

3GPP release timeline: Path from 4G to 5G



LTE-A Pro based on existing LTE-A Rel-13

The 3rd Generation Partnership Project (3GPP) is a collaboration between groups of telecommunications associations, known as the Organizational Partners.

Trends of 5G mobile networks technologies

Technologies to enhance the radio interface

- Advanced modulation, coding and multiple access schemes
- Advanced antenna and multi-site technologies (active antenna system, massive MIMO)
- Physical layer enhancements and interference handling for small cell
- **Flexible spectrum usage** (**Licensed shared access, LSA)**

Technologies to support wide range of emerging services

- **Technologies to support the proximity services (D2D)**
- **Technologies to support M2M services**

Technologies to enhance user experience

- **QoS enhancement**
- Mobile video enhancement

Technologies to improve network energy efficiency

- Network-level power management
- Energy-efficient network deployment

Terminal technologies

- Interference cancellation and suppression

Network Technologies (small cells, **ultra dense network**, Cloud-RAN)

Technologies to enhance privacy and security

Research activities in Applied Mathematics & Communications Technology Institute (RUDN)

- Research and development of models and methods for planning, analysis, and utilization of wireless 5G systems and beyond.
- Advanced research on mobile 5G networks and emerging IoT applications.
- Development of mathematical techniques and tools for conducting quality-centric evaluation for the IoT infrastructure with the emphasis on device mobility.
- Modeling and numerical analysis for advanced telecom related problems and mobile technologies.
- Research and development of computational methods and descriptive models of complex systems.
- Development of advanced probabilistic techniques to improve reliability and efficiency of device interaction in 5G networks and IoT services.
- Constructing new tools for traffic modeling in next-generation networks as well as proposing efficient resource management schemes.

Ongoing Projects

- Resource allocation in wireless networks with random resource requirements
- Interruption probability analysis for Licensed Shared Access (LSA) regulatory framework
- Stochastic geometry models and SIR analysis in D2D wireless networks
- Modelling users’ mobility

Ongoing Projects: Project 1

- **Resource allocation in wireless networks with random resource requirements**
- Interruption probability analysis for Licensed Shared Access (LSA) regulatory framework
- Stochastic geometry models and SIR analysis in D2D wireless networks
- Modelling users' mobility

Project 1 publications

1. Pyattaev A., Johnsson K., Surak A., Florea R., Andreev S., Koucheryavy Y. “Network-assisted D2D communications: Implementing a technology prototype for cellular traffic offloading”, Wireless Communications and Networking Conference (WCNC), pp 3266-3271, 2014.
2. V. Naumov, K. Samouylov, E. Sopin, S. Andreev. Two approaches to analyzing dynamic cellular networks with limited resources. Proc. 6th Int. Congress on Ultra Modern Telecommunications and Control Systems (ICUMT), St. Petersburg, Russia, 6-8 Oct. 2014, 485 - 488.
3. Naumov V.A., Samouylov K.E, Samuylov A.K. “On total amount of resources occupied by customers”. Automation and Remote Control, vol.4, 2015.
4. V. Naumov, K. Samuoylov. On relationship between queuing systems with resources and Erlang networks, Informatics and Applications, 2016, Vol. 10, No. 3, 9-14.
5. V. Naumov, K. Samouylov. Analysis of multi-resource loss system with state dependent arrival and service rates. Probability in the Engineering and Informational Sciences. 2017, Vol. 31, No. 4, 413-419.
6. Naumov V., Samouylov K., Yarkina N., Sopin E., Andreev S., Samuylov A. LTE performance analysis using queuing systems with finite resources and random requirements. International Congress on Ultra Modern Telecommunications and Control Systems, Lisbon; Portugal (18-20 October 2016). No. 7382412. P. 100-103.
7. V. Petrov, D. Solomitckii, A. K. Samuylov, M.A. Lema, M. Gapeyenko, D. Moltchanov, S. Andreev, V. Naumov, K.E. Samouylov, M. Dohler, Y. Koucheryavy. Dynamic multi-connectivity performance in ultra-dense urban mmWave deployments. IEEE J. Selected Areas in Communications, 2017 , Vol. 35, No.9, 2038-2055.

Loss systems with random resource requirements (~2010) (1/2)

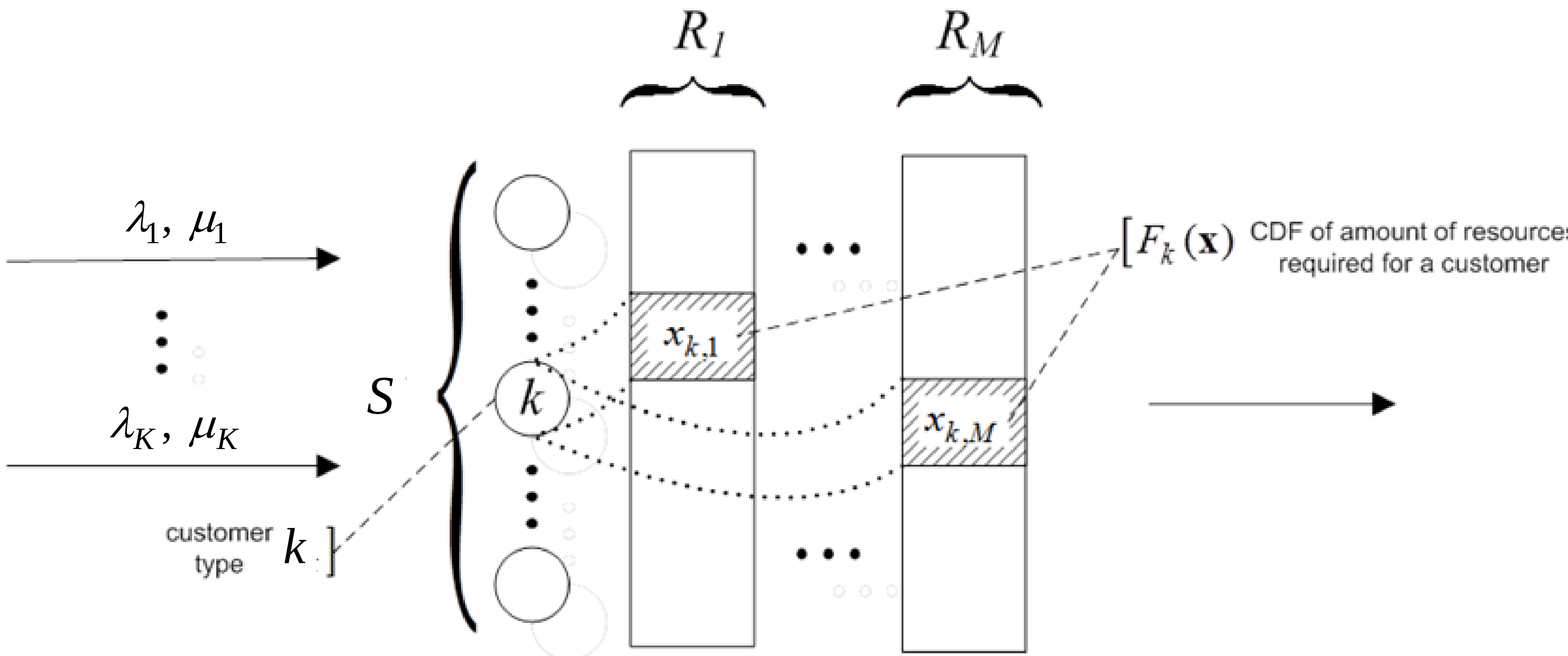
- Acquisition of multiple *resources of different types:*
  - There are R_m units of resources of type m, $\boldsymbol{R} = (R_1, \dots, R_M)$
  - The i-th customer of class k requires to hold $r_{km}(i)$ units of resources of type m.
  - Resources demands $r_k(i) = \left(r_{k1}(i), \dots, r_{kM}(i)\right), i = 1,2, \dots$ of class k customers are nonnegative random vectors with cumulative distribution functions $F_k(\boldsymbol{X})$
- The set of feasible states is given by

$$X = \{(\boldsymbol{n}, \gamma_1, \dots, \gamma_K) | \boldsymbol{n} \in \mathbb{N}^K, \gamma_k \in \mathbb{R}_+^M,$$
$$k = 1,2, \dots K, \sum_{k=1}^{K} \gamma_k \le \boldsymbol{R}, \sum_{k=1}^{K} n_k \le S\}$$

$\boldsymbol{n} = (n_1, \dots, n_K)$ – population vector

$\gamma_k = (\gamma_{k1}, \dots, \gamma_{kM})$ – vector of resources occupied by class k customers

Loss systems with random resource requirements (~2010) (2/2)



Loss systems with random resource requirements

- Cumulative distribution functions of the stationary distribution are given by

$$P_{\boldsymbol{n}}(\boldsymbol{x}_1, \dots, \boldsymbol{x}_K) = \frac{1}{G}\prod_{k=1}^{K}\frac{\rho_k^{n_k}}{n_k!}F_k^{*n_k}(\boldsymbol{x}_k), (\boldsymbol{n}, \boldsymbol{x}_1, \dots, \boldsymbol{x}_K) \in X \qquad (7)$$

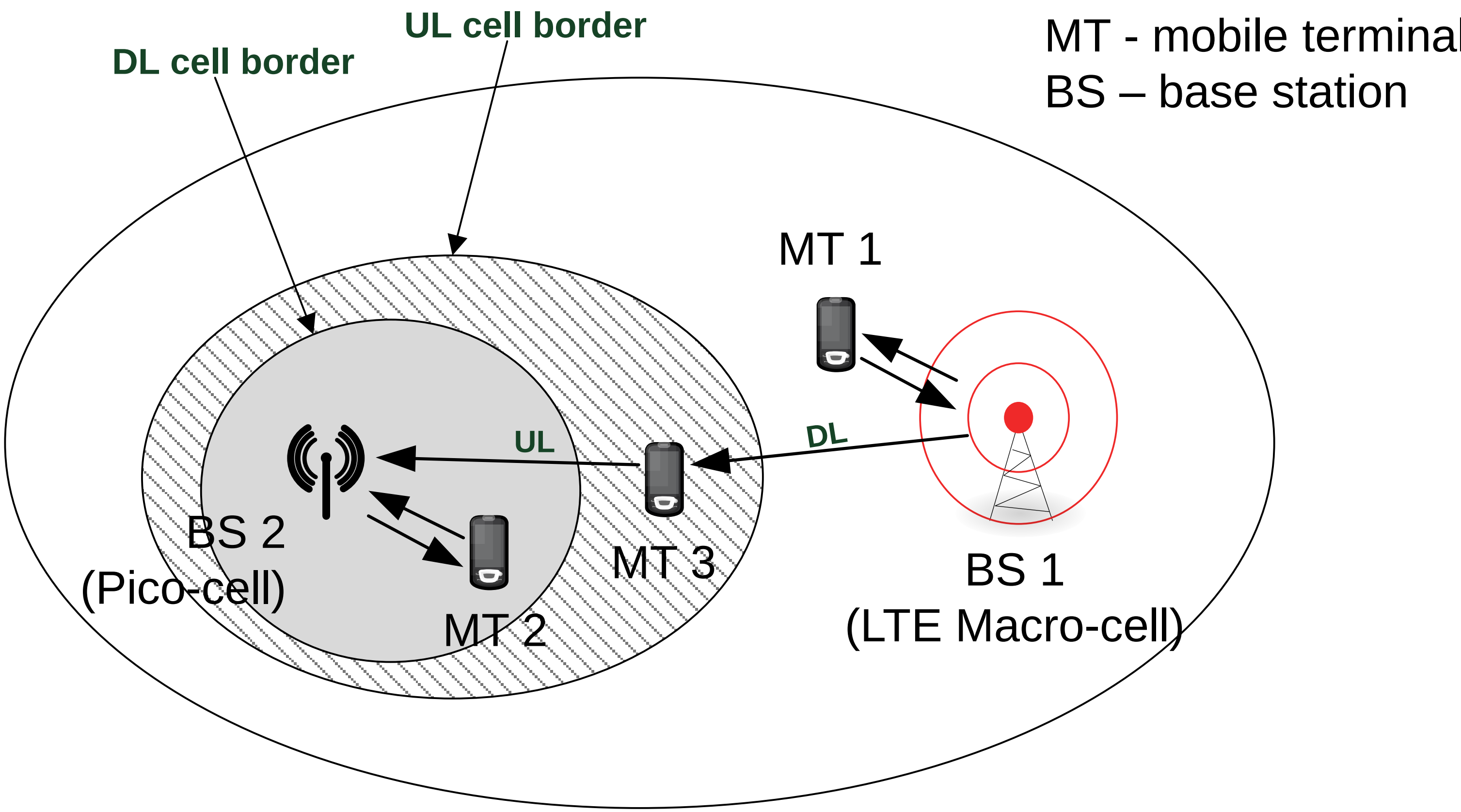
$$G = \sum_{n_1+\cdots+n_K \le S}(F_1^{*n_1} * \cdots * F_k^{*n_k})(\boldsymbol{R})\frac{\rho_1^{n_1} \dots \rho_k^{n_k}}{n_1! \dots n_k!}$$

$*$ - convolution symbol

- Blocking probability of class k customers: $B_k = 1 - \frac{G_k}{G}$, (8)

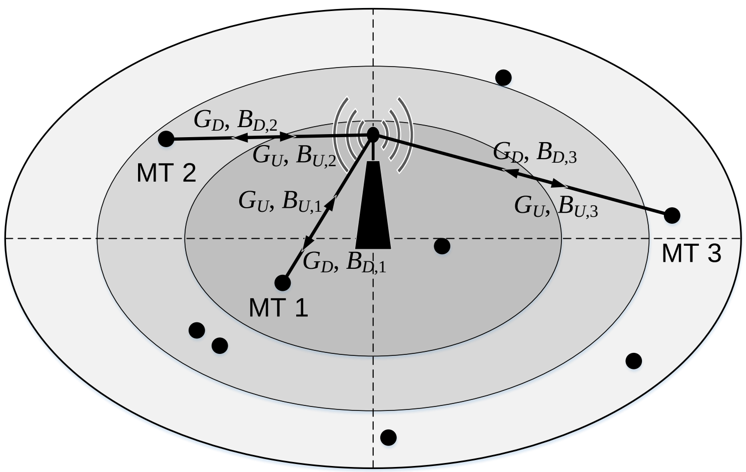
$$G_k = \sum_{n_1+\cdots+n_K \le S}\left(F_1^{*n_1} * \cdots * F_k^{*(n_k+1)} * \cdots * F_K^{*n_K}\right)(\mathbf{R})\frac{\rho_1^{n_1} \cdots \rho_K^{n_K}}{n_1! \cdots n_K!}$$

Uplink/downlink decoupling in LTE wireless network



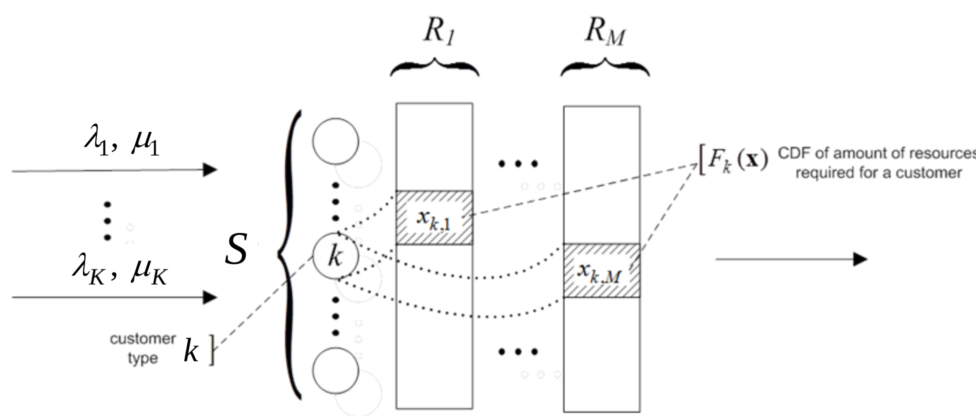
Elshaer, H., Boccardi, F., Dohler, M., Irmer, R. “Downlink and Uplink Decoupling: A disruptive architectural design for 5G networks”, Global Communications Conference (GLOBECOM), pp. 1798 – 1803, 2014.

Radio resources allocation in LTE wireless network



Call type	Recommended download speed	Recommended upload speed	%
Video calling (high quality)	400 kbit/s	400 kbit/s	40
Video calling (HD)	1.2 Mbit/s	1.2 Mbit/s	30
Group video (3 users)	2 Mbit/s	500 kbit/s	20
Group video (5 users)	4 Mbit/s	500 kbit/s	10

G_U и G_D –uplink/downlink rates
$B_{U,i}$ и $B_{D,i}$ – number of PRB for *i*-th session,
$B_{D,3} > B_{D,2} > B_{D,1}$ *и* $B_{U,3} > B_{U,2} > B_{U,1}$



Loss systems with random resource requirements

Loss systems with dependent resource requirements and service time

- Acquisition of multiple resources of different types:
  - There are R_m units of resources of type m, $\boldsymbol{R} = (R_1, \dots, R_M)$
  - The i-th customer of class k requires to hold $r_{km}(i)$ units of resources of type m.
- Service times $\beta_k(i)$ and resource demands $\mathbf{r}_k(i)$ of class *k* customers, *i* =1,2,…, have $F_k(x)$ joint cumulative distribution functions $H_k(t,\mathbf{x}) = P\{\beta_k(j) \le t, \mathbf{r}_k(j) \le \mathbf{x}\}$
- Stationary distribution $P_{\mathbf{n}}(\mathbf{x}_1,\dots,\mathbf{x}_K)$ of the system is exactly the same as for the system, in whch service times and resource demands are independent, service times are exponentially distributed with the rate $\mu_k = 1/b_k$ and probability distribution functions of resource requirements $F_k(\mathbf{x})$ given by

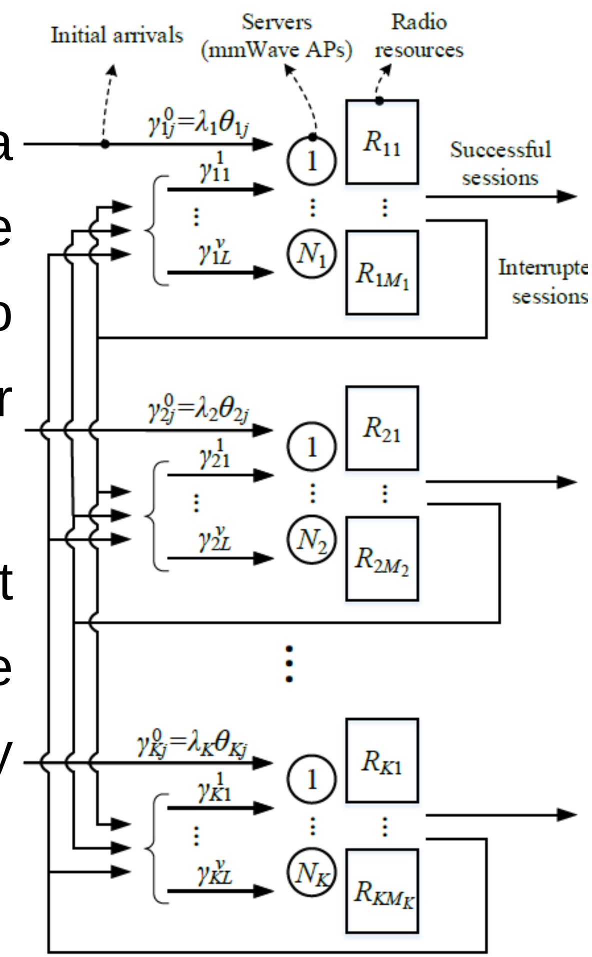
$$b_k = \lim_{\substack{x_1 \to \infty \\ \dots \\ x_K \to \infty}} \int_0^\infty t H_k(dt,\mathbf{x}) \qquad F_k(\mathbf{x}) = \frac{1}{b_k}\int_0^\infty t H_k(dt,\mathbf{x})$$

Loss systems with positive and negative resource requirements

- Resource demands $r_k(i) = \left(r_{i1}(i), \dots, r_{kM}(i)\right), i = 1,2, \dots$ of class k customers are random ~~nonnegative~~ vectors with cumulative distribution function $F_k(\mathbf{x})$
- Acquisition of a *positive* quantity of a resource means *subtraction* of this quantity from the pool of available resources
- Acquisition of a *negative* quantity of a resource means *addition* of this quantity from the pool of available resources
- A customer with negative resource demand can leave the system only if the resource that was added to the pool of available resources can be picked up without disrupting the service of other calls

Loss networks with random resource requirements and signals (2017)

- Network contains customers and signals.
- Arriving signal interrupts the service of a customer and forces a customer to leave the network, or to move instantaneously to another loss system where the customer requests new service.
- If the service of a customer was not interrupted, the customer leaves the network and is considered as successfully served.



Ongoing Projects: Project 2

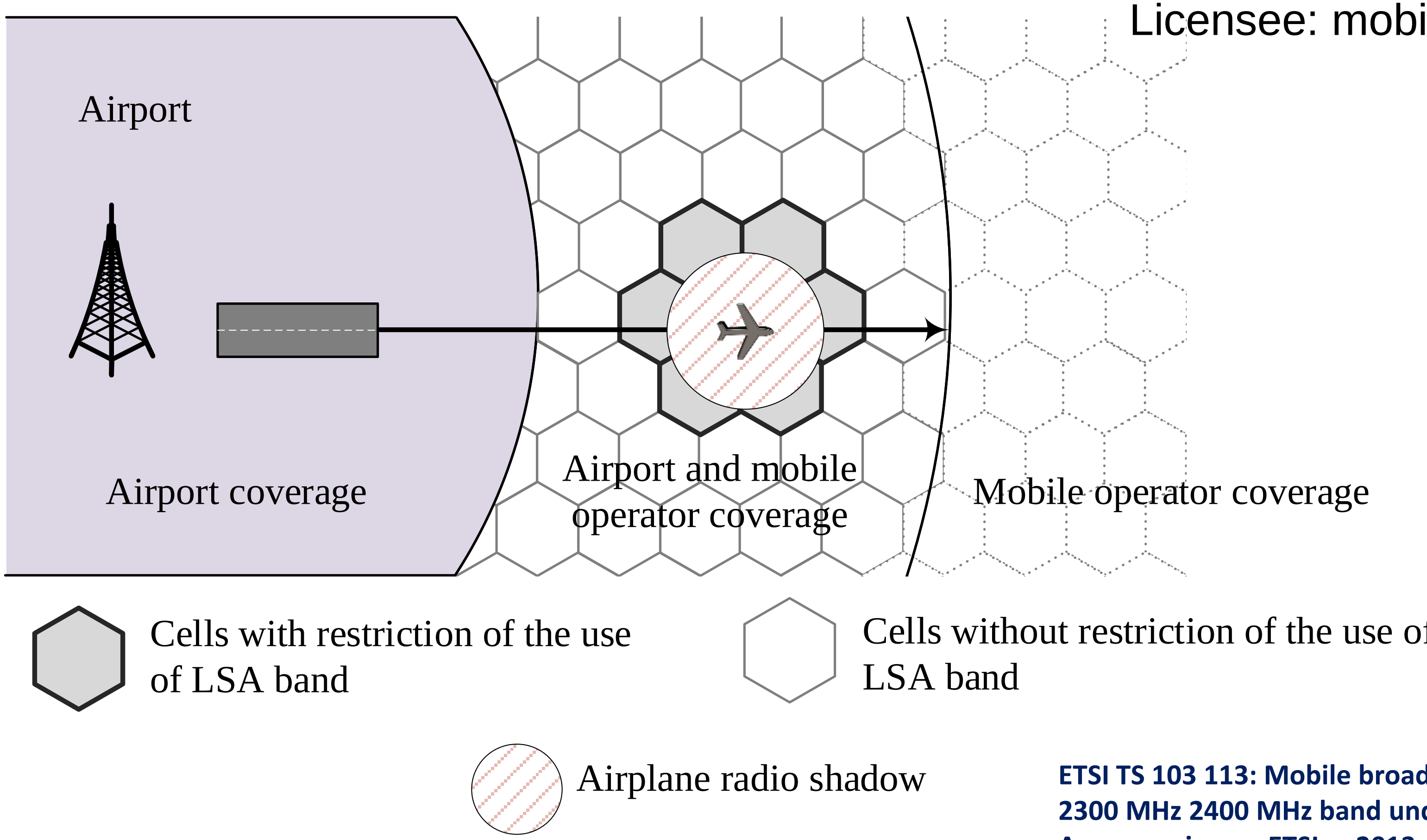
- Resource allocation in wireless networks with random resource requirements
- **Service Failure and Interruption probability analysis for Licensed Shared Access (LSA) regulatory framework**
- Stochastic geometry models and SIR analysis in D2D wireless networks
- Modelling users' mobility

Project 2 publications

1. Borodakiy V.Y., Samouylov K.E., Gudkova I.A., Ostrikova D.Y., Ponomarenko A.A., Turlikov A.M., and Andreev S.D. Modeling unreliable LSA operation in 3GPP LTE cellular networks // Proc. of the 6th International Congress on Ultra Modern Telecommunications and Control Systems ICUMT-2014 (October 6–8, 2014, St. Petersburg, Russia). – IEEE. – 2014. – P. 490–496.
2. Gudkova I.A., Samouylov K.E., Ostrikova D.Y., Mokrov E.V., Ponomarenko-Timofeev A.A., Andreev S.D., and Koucheryavy E.A. Service failure and interruption probability analysis for Licensed Shared Access regulatory framework // Proc. of the 7th Int. Congress on Ultra Modern Telecommunications and Control Systems ICUMT-2015 (October 3–5, 2012, St. Petersburg, Russia). – IEEE. – 2015.
3. Mokrov E.V., Ponomarenko-Timofeev A.A., Gudkova I.A., Andreev S.D., and Samouylov K.E. Modeling a load balancing scheme between primary licensed and LSA frequency bands in 3GPP LTE networks // Proc. of the IX International Workshop "Applied Problems in Theory of Probabilities and Mathematical Statistics related to modeling of information systems" APTP+MS-2015 (August 10–13, 2015, Tampere, Finland). – Finland, Tampere. – 2015. – P. 54–57.
4. Gudkova I., Samouylov K., Ostrikova D., Mokrov E., Ponomarenko-Timofeev A., Andreev S., Koucheryavy Y. Service failure and interruption probability analysis for Licensed Shared Access regulatory framework. / International Congress on Ultra Modern Telecommunications and Control Systems and Workshops, Lisbon; Portugal (18-20 October 2016). - No. 7382416. - C. 123-131.

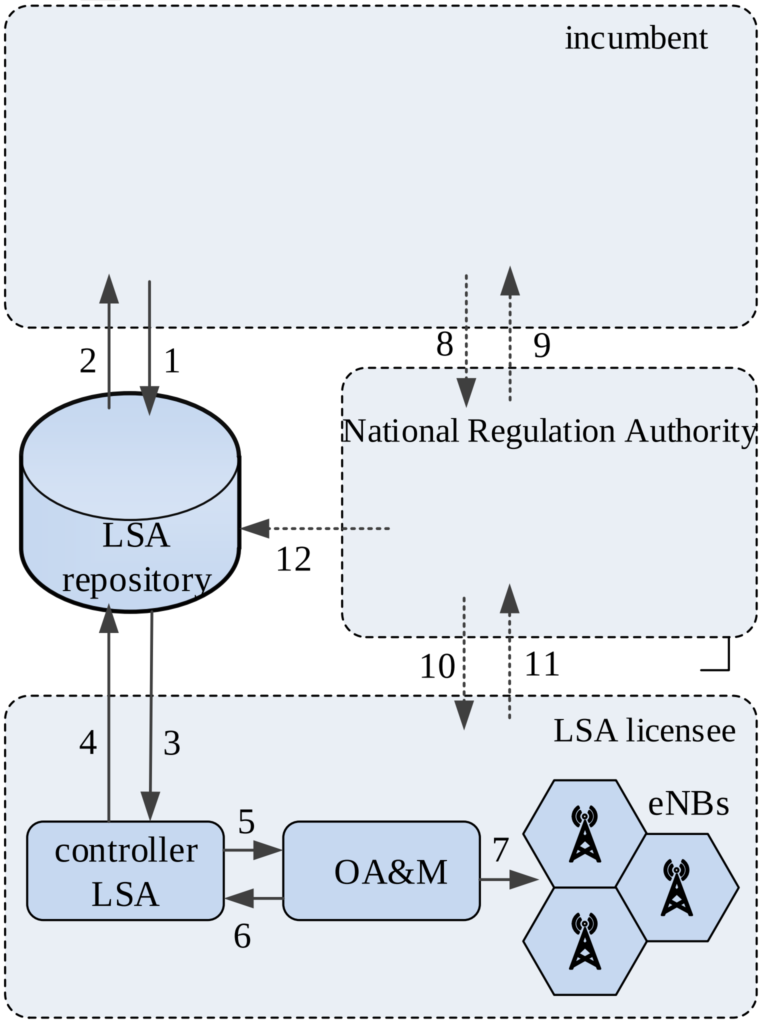
Example of the Licensed Shared Access (LSA)

Incumbent: airport
Licensee: mobile operator



ETSI TS 103 113: Mobile broadband services in the 2300 MHz 2400 MHz band under Licensed Shard Access regime. – ETSI. – 2013.

Licensed Shared Access (LSA) architecture



Architecture elements:

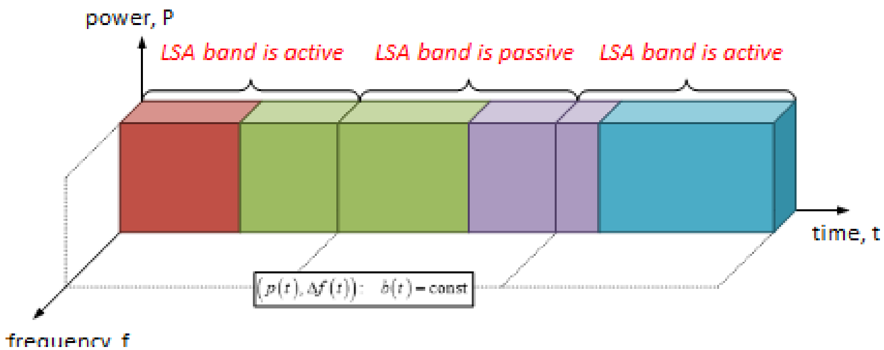
• **LSA Repository**
This database contains the relevant information on spectrum use by the incumbent (in the spatial, frequency and time domains). Furthermore, the incumbent may choose to take steps ensuring that its confidentiality and information sensitivity requirements are met.

•**LSA Controller**
The LSA Controller computes LSA spectrum availability in the spatial, frequency and time domains based on rules built upon LSA rights of use and information on the incumbent's use provided by the LSA Repository.

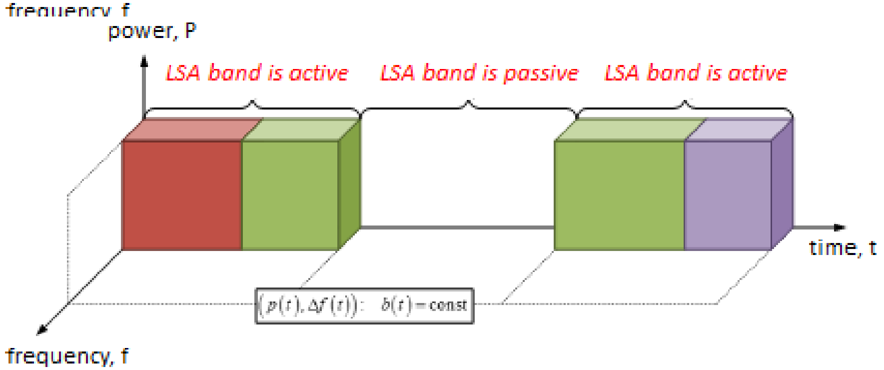
Network OA&M
(Operations, Administration and Maintenance)
OA&M entity performs the actual management of the LSA spectrum by issuing the radio resource management (RRM) commands based on the information received from the LSA controller.

ETSI TS 103 113: Mobile broadband services in the 2300 MHz 2400 MHz band under Licensed Shard Access regime. – ETSI. – 2013.

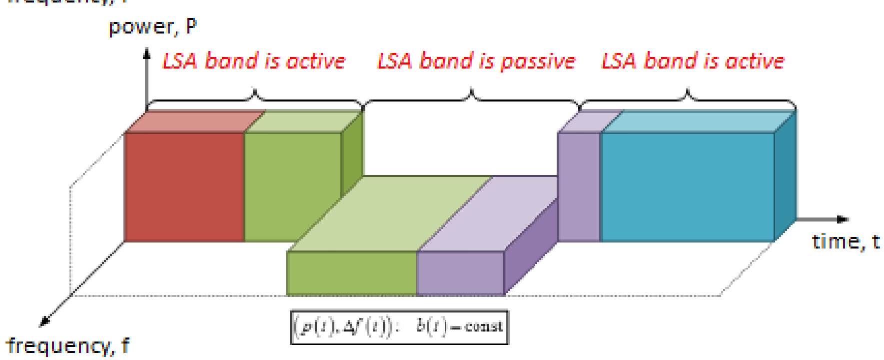
Policies of LSA



Ignore policy: LSA band is always available to the licensee, in other words no coordination is to be introduced between the LSA incumbent and licensee.

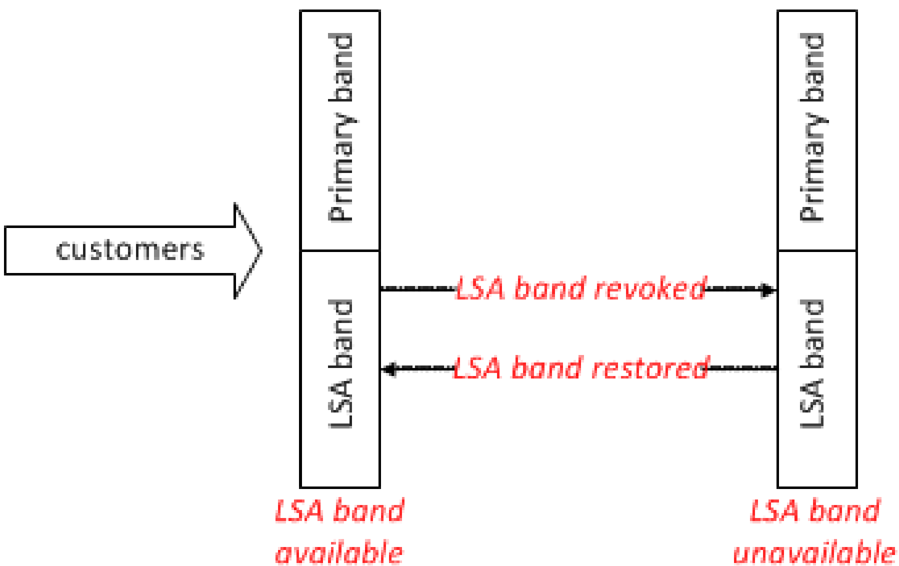


Shutdown policy: LSA band is fully unavailable to the licensee. All the BSs whose UEs have a chance to cause interference on LSA bands are ”powered off”.



Limit power policy: LSA band is available, though it is used with reduced power. All the BSs are forced to reduce the corresponding UE’s uplink power whenever instructed.

Model scenarios



When the LSA band is being revoked:

- block the customers using LSA band
- transfer customers from LSA band to the primary licensed band

When the LSA band is being recovered:

- transfer some customers from the primary licensed band to the LSA band
- not to transfer customers

When there a position vacates on the primary band:

- transfer customers from LSA band to primary licensed band
- not to transfer customers

Reliability models and queuing systems with unreliable servers

Reliability models

In stable environment	$< G_n / G / m >$: Vishnevsky V.M, Rykov V.V.
In random environment	$< G_n / G / m \ (RE) >$: Vishnevsky V.M., Rykov V.V, Yechiali U., D’Auria B., and other authors

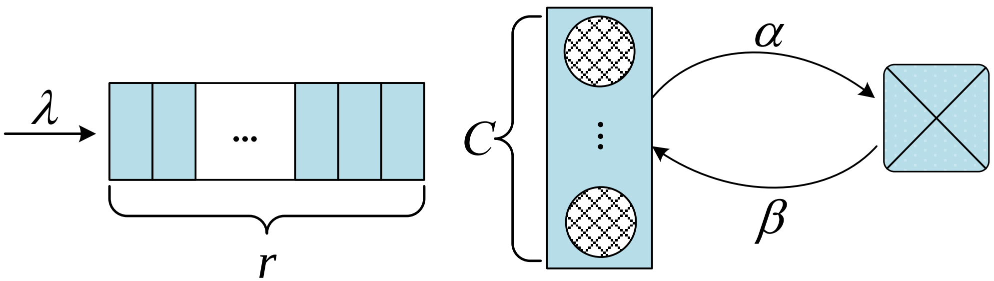
Queuing systems with unreliable servers

<table>
<tr><td colspan="2" rowspan="3">Single-server queueing system</td><td>Simultaneous failures</td><td>Independent failures</td><td>Group failures</td></tr>
<tr><td colspan="3">$G / G / 1 / \infty$: Afanasyeva L.G., Vishnevsky V.M., Klimov G.P., Pechinkin A.V., Efrosinin D.V., Klimenok V.I., Gaver D.P., Kharoufeh J.P., Li Q.-L., Wang J., and other authors</td></tr>
<tr><td colspan="3">$G / G / 1 / r$: Basharin G.P., Samouylov K.E., Kovalenko A.I, Yang D.-Y.</td></tr>
<tr><td rowspan="2">Multi-server queueing system</td><td>$G / G / n / \infty$</td><td>Monemian M. [3]
Rao S.S.</td><td>Emelyanov G.V.
Pechinkin A.V.
Chaplygin V.V.
Liu G.-S.</td><td>—</td></tr>
<tr><td>$G / G / n / r$</td><td>Pechinkin A.V. [1]</td><td>Pechinkin A.V.
Mikadze I.S.</td><td>Pechinkin A.V.</td></tr>
<tr><td>Hybrid queueing system</td><td colspan="4">$G / G / 2 / \infty$: Vishnevsky V.M. [2], Semenova O.V. [2]</td></tr>
</table>

Performance measures

- **For operator**
  - Non-interrupting probability for all users, i.e. probability that if LSA band fails then there is no need to interrupt service, i.e., the operator does not need to discontinue service for any of its users; none of the users will suffer from performance degradation.
  - Interrupting probability for at least one user, i.e. probability that if the LSA band fails than at least one user using it will be interrupted
  - Probability that the LSA band is unavailable.
  - The mean number of user requests in the queue, i.e. the average number of users waiting for their service to start or for it to continue.
- **For user**
  - Probability that if the LSA band fails then for a target user the service will not be interrupted
  - Probability that if the LSA band fails then for a target user the service will not interrupted
  - Probability that a user request is blocked.

Model of LSA framework with unique band



C	Number of servers
λ	Customer arrival rate
μ^{-1}	Average customer service time
α	Servers failure rate
β	Servers recovery rate
n	Number of occupied servers (the number of serving users)
m	Number of users waiting to receive service

Markov process

$$\{(N(t), M(t)),\ t \ge 0\}$$

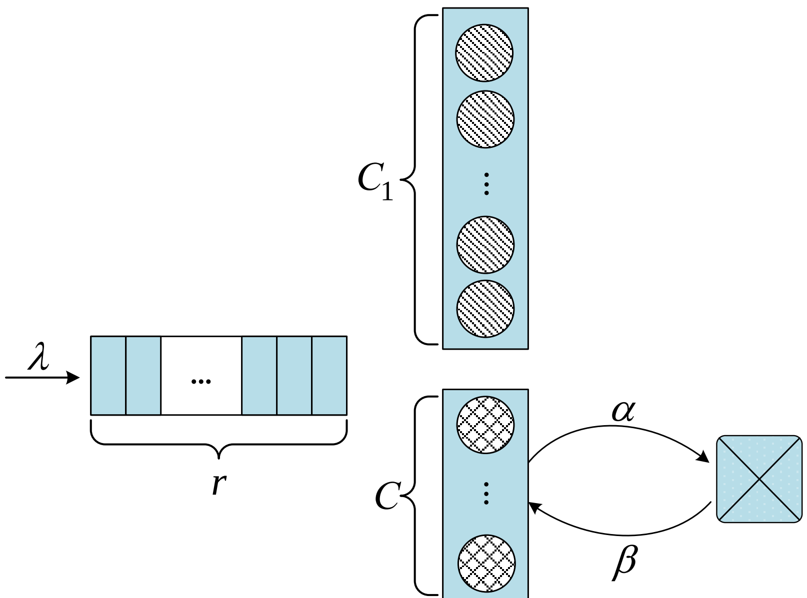
State space

$$\mathcal{Y} = \{(n,m) : (n,0),\ n = 0,\dots,C;\ (C,m),\ m = 1,\dots,r-C;\ \ (0,m),\ m = 1,\dots,r\}$$

Stationary distribution

$$p(n,m) = \lim_{t\to\infty} P(N(t) = n,\ M(t) = m),\ (n,m) \in \mathcal{Y}$$

Model of LSA framework with primarily bands



C_1	Number of reliable servers
C	Number of unreliable servers
λ	Customer arrival rate
μ^{-1}	Average customer service time
α	Servers failure rate
β	Servers recovery rate
n_1	Number of occupied reliable servers (number of users on reliable servers)
n	Number of occupied unreliable servers (number of users on unreliable servers)
m	Number of users waiting to receive service
s	State of unreliable servers: $s=1$ if unreliable servers are operational, $s=0$ if unreliable servers are unavailable

Markov process

$$\{(N_1(t), N(t), M(t), S(t)),\ t \ge 0\}$$

State space

$$\mathscr{X} = \{n_1 = 0,\dots,C_1,\ n = 0,\dots,C,\ m = 0,\ s = 1;\ \ n_1 = C_1,\ n = C,\ m = 1,\dots,r-C,\ s = 1;$$
$$n_1 = 0,\dots,C_1,\ n = 0,\ m = 0,\ s = 0;\ \ n_1 = C_1,\ n = 0,\ m = 1,\dots,r,\ s = 0\}$$

Stationary distribution

$$p(n_1,n,m,s) = \lim_{t\to\infty} P(N_1(t) = n_1,\ N(t) = n,\ M(t) = m,\ S(t) = s),\ (n_1,n,m,s) \in \mathscr{X}$$

Performance measures

- Probability that when the unreliable servers fail, at least one customer service will be interrupted

$$I_1 = \sum_{n=1}^{C} \sum_{n_1=C_1-n+1}^{C_1} \frac{\alpha}{\alpha+\lambda+(n+n_1)\mu} p(n_1,n,0,1) + \sum_{m=1}^{r-C-1} \frac{\alpha}{\alpha+\lambda+(C+C_1)\mu} p(C_1,C,m,1) + \frac{\alpha}{\alpha+(C+C_1)\mu} p(C_1,C,r-C,1),$$

- Probability that when the unreliable servers fail then for a target customer the service will be interrupted

$$I_2 = \sum_{n=1}^{C} \sum_{n_1=C_1-n+1}^{C_1-1} \frac{n-C_1+n_1}{n} \cdot \frac{\alpha}{\alpha+\lambda+(n+n_1)\mu} p(n_1,n,0,1) + \sum_{n=1}^{C} \frac{\alpha}{\alpha+\lambda+(n+C_1)\mu} p(C_1,n,0,1) +$$

$$+ \sum_{m=1}^{r-C-1} \frac{\alpha}{\alpha+\lambda+(C+C_1)\mu} p(C_1,C,m,1) + \frac{\alpha}{\alpha+(C+C_1)\mu} p(C_1,C,r-C,1),$$

- Probability that when the unreliable servers fail, no customer service will be interrupted

$$E_1 = \sum_{n=0}^{C} \sum_{n_1=0}^{C_1-n} \frac{\alpha}{\alpha+\lambda+(n+n_1)\mu} p(n_1,n,0,1),$$

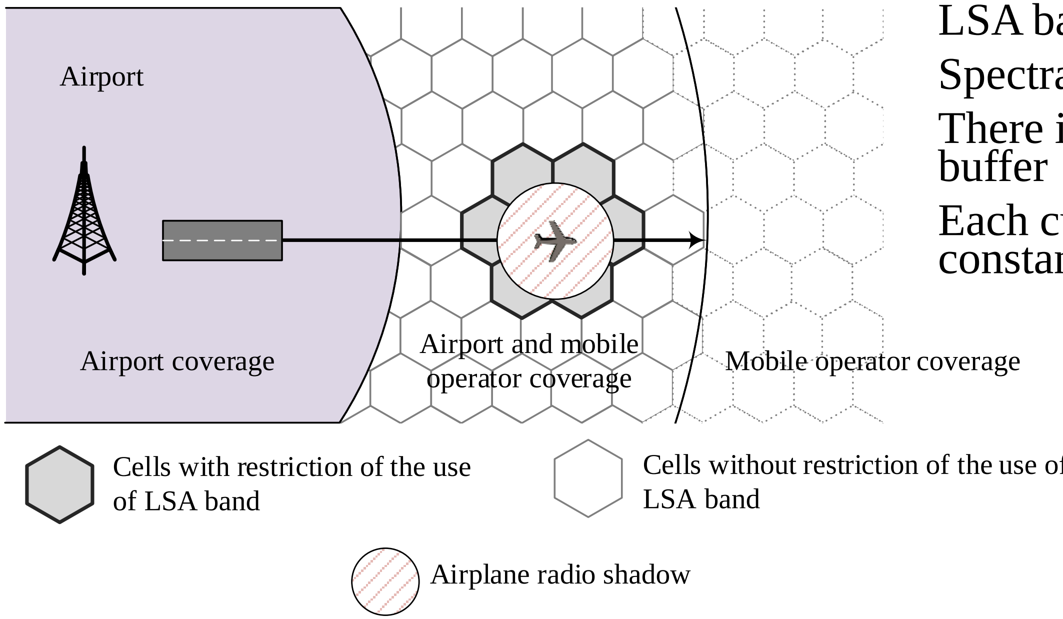
- Probability that when the unreliable servers fail then for a target customer the service will not be interrupted

$$E_2 = \sum_{n=1}^{C} \sum_{n_1=0}^{C_1-n} \frac{\alpha}{\alpha+\lambda+(n+n_1)\mu} p(n_1,n,0,1) + \sum_{n=1}^{C} \sum_{n_1=C_1-n+1}^{C_1-1} \frac{C_1-n_1}{n} \cdot \frac{\alpha}{\alpha+\lambda+(n+n_1)\mu} p(n_1,n,0,1),$$

- Blocking probability

$$B = p(C_1,0,r,0) + p(C_1,C,r-C,1),$$

Numerical analysis: Input data



Main bandwidth is 10 MHz.

LSA bandwidth is 5 MHz.

Spectral efficiency in LTE is 4 bits/Hz

There is place for 30 customers in the buffer

Each customer generates a session with constant bit rate of 250 KBps

Using this data we can calculate how many customers our system can accommodate:

$$C_1 = \frac{(\text{Bandwidth})\cdot(\text{Spectral efficiency})}{(\text{bitrate})} = \frac{10^7 \cdot 4}{250 \cdot 8 \cdot 10^3} = 20,\ C_2 = \frac{5 \cdot 10^6 \cdot 4}{250 \cdot 8 \cdot 10^3} = 10$$

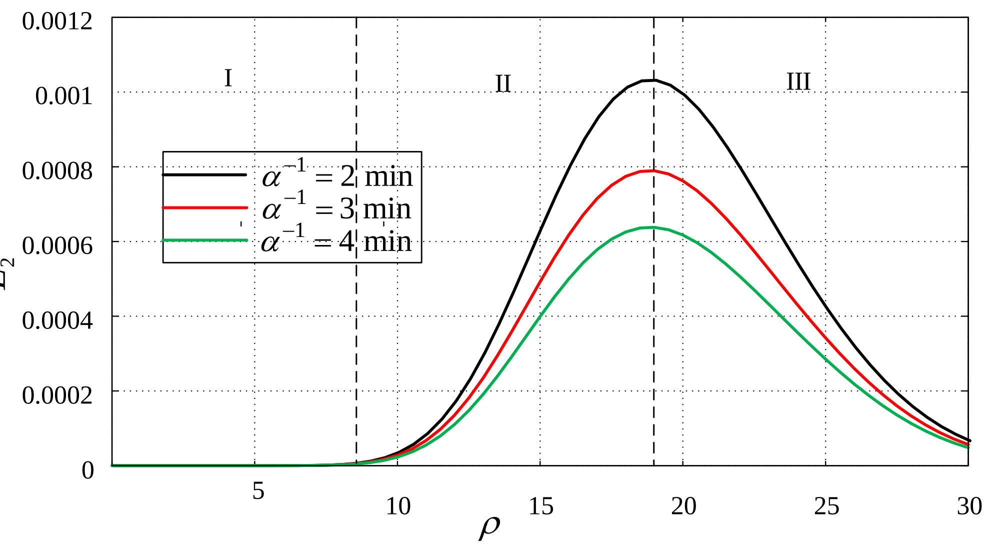
Average time to finish transmitting one session is $\mu^{-1} = 15\text{s}$.

Average time between sessions for 1 customer is varied

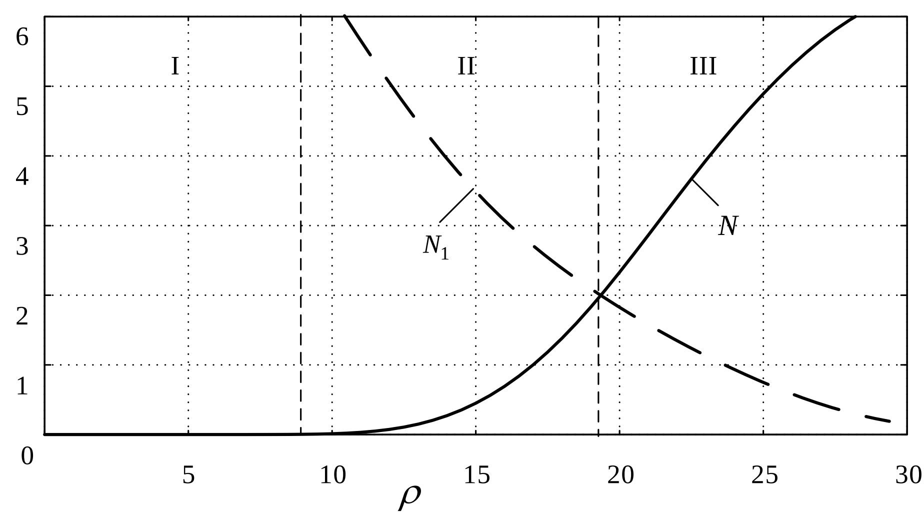
LSA band is requested by the airport in average every $\alpha^{-1} = 120\text{s}$ (180s, 240s) for an average time of $\beta^{-1} = 60\text{s}$

Numerical analysis

Probability that when the unreliable servers fail then for a target customer the service will not be interrupted



Mean number of users on LSA band and mean number of vacant places on individual band



- Mean number of users on LSA band

$$N = \sum_{n_1=0}^{C_1} \sum_{n=0}^{C} n p(n_1, n, 0, 1) + C \sum_{m=1}^{r-C} p(C_1, C, m, 1)$$

- Mean number of vacant places on individual band

$$N_1 = \sum_{n_1=0}^{C_1} \sum_{n=0}^{C} (C_1 - n_1) p(n_1, n, 0, 1)$$

Ongoing Projects: Project 3

- Resource allocation in wireless networks with random resource requirements
- Interruption probability analysis for Licensed Shared Access (LSA) regulatory framework
- **Stochastic geometry models and SIR analysis in D2D wireless networks**
- Modelling users' mobility

Project 3 publications

1. Samuylov, A., Yu. Gaidamaka, D. Moltchanov, S. Andreev, and Y. Koucheryavy. 2015. Random triangle: A baseline model for interference analysis in heterogeneous networks. IEEE Trans. Veh. Technol. 65(8):6778-6782. doi:10.1109/TVT.2016.2596324.
2. Samuylov, A., A. Ometov, V. Begishev, R. Kovalchukov, D. Moltchanov, Yu. Gaidamaka, K. Samouylov, S. Andreev, and Y. Koucheryavy. 2015. Analytical performance estimation of network-assisted D2D communications in urban scenarios with rectangular cells. Trans. Emerg. Telecomm. Technol. 28(2):2999-1-2999-15. doi:10.1002/ett.2999.
3. Samuylov A., D. Moltchanov, Yu. Gaidamaka, V. Begishev, R. Kovalchukov, P. Abaev, S. Shorgin. 2016. SIR analysis in square-shaped indoor premises // Proc. of the 30th European Conference on Modelling and Simulation ECMS-2016 (May 31 – June 03, 2016, Regensburg, Germany). – Germany, Digitaldruck Pirrot GmbH. – Pp. 692-697. doi: 10.7148/2016-0692
4. Abaev P., Gaidamaka Yu., Samouylov K., Shorgin S. 2016. Tractable distance distribution approximations for hardcore processes // V.M. Vishnevskiy et al. (Eds.): DCCN 2016, CCIS 678, pp. 98–109, 2016. doi: 10.1007/978-3-319-51917-3 10
5. Etezov, Sh., Yu. Gaidamaka, K. Samuylov, D. Moltchanov, A. Samuylov, S. Andreev, and E. Koucheryavy. 2016. On Distribution of SIR in Dense D2D Deployments. 22nd European Wireless conference (EW’2016), May 18-20, 2016, Oulu, Finland. P. 333-337.

D2D communication

Device-to-device (D2D) communication - direct communication between nearby mobile devices [3GPP LTE Release 12].

The main motivation to define D2D communication in LTE Rel.12 is for Public Safety authorities such as police, firefighters and ambulances.

General advantages

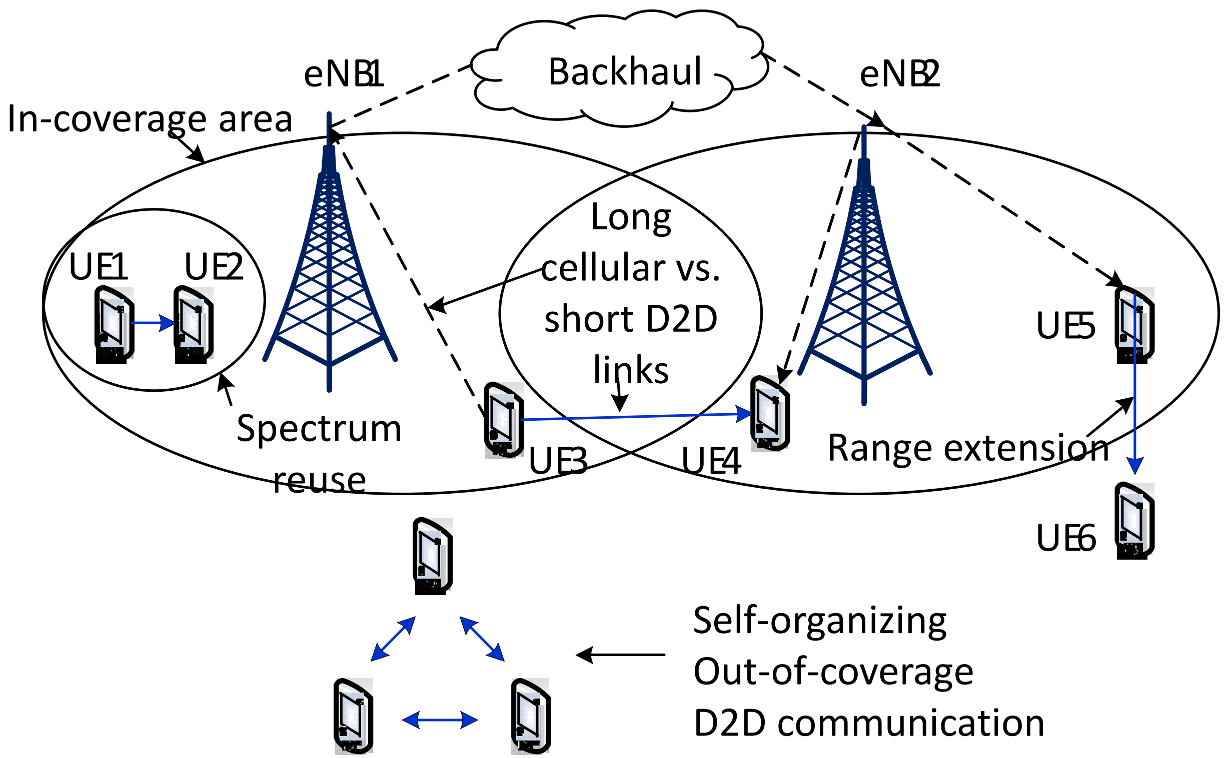
- improve spectrum utilization
- improve overall throughput and performance
- improve energy consumption
- enable new peer-to-peer and location-based applications and services

Advantages related to public safety

- fallback public safety networks that must function when cellular networks are not available or fail
- closing the evolution gap of safety networks to LTE

A major impediment - interference.

D2D use cases



Benefits:

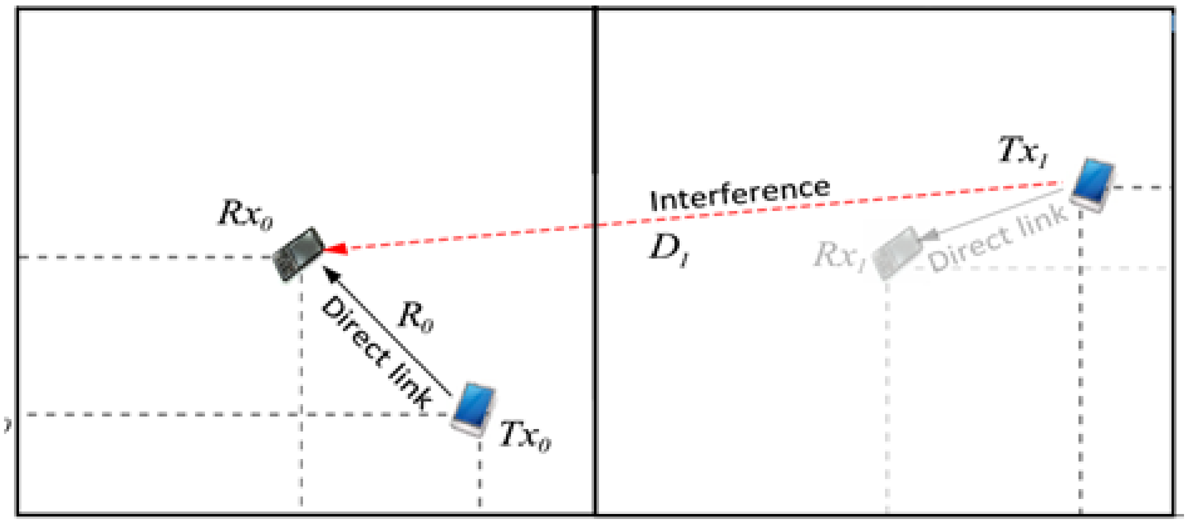
- high data rates,
  low end-to-end delay
  due to the shortrange
  direct communication
- resource-efficiency
  due to the direct
  communication
  instead of routing
  through an evolved
  Node B (eNB)
  and the core network
- energy saving
- cellular traffic offloading
- alleviating congestion

J. Andrews,. X. Lin, A. Ghosh, and R. Ratasuk. An Overview of 3GPP Device-to-Device Proximity Services, IEEE Communications Magazine, April 2014.

Problem statement: fixed devices

Target pair

"receiver – transmitter"



Interfering pair

"receiver – transmitter"

<u>Interference</u> in wireless networks refers to the interaction of signals transmitted by different sources (mobile devices) on the same radio channel or nearby channels.

The interference causes distortion of the signal of the source due to the influence of the signal from an external source.

Signal-to-Interference Ratio (SIR) as the ultimate metric

$$SIR = \frac{S}{\sum_{i=1}^{N} I_i} \text{ – Signal-to-Interference Ratio} \qquad (1)$$

$S = S(R_0) = g_0 R_0^{-\gamma_0}$ – the power of the useful signal

$I_i = I_i(D_i) = g_i D_i^{-\gamma_i}$ – the interference power of the *i*-th transmitter

g_i – basic *i*-th transmitter power

γ_i – the propagation exponent for *i*-th transmitter

N – number of transmitters

Goal: to obtain *SIR* cumulative distribution function

System model

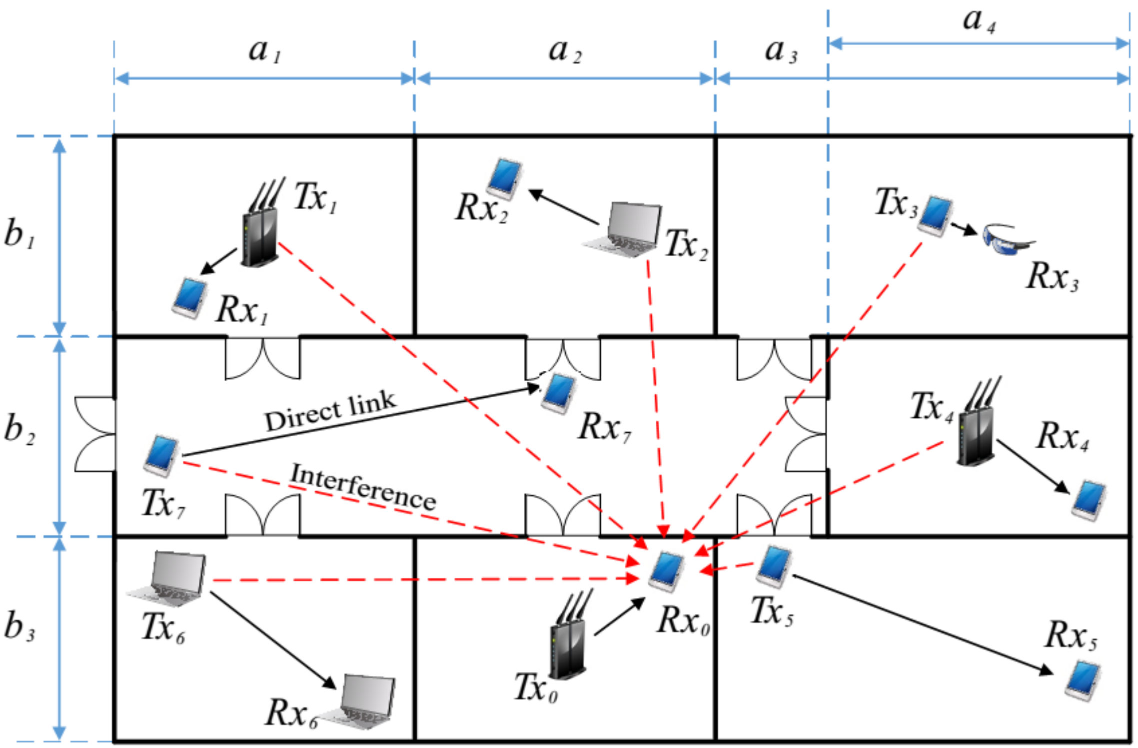


Figure 2. The considered D2D deployment in a city mall

Rx_i – receiving device
Tx_i – transmitting device
a_i, b_i – clusters sides lengths

$$SIR = \frac{S(R_0)}{\sum_{i=1}^{N} I(D_i)} = \frac{gR_0^{-\gamma_1}}{g\sum_{i=1}^{N} D_i^{-\gamma_j}} = \frac{R_0^{-\gamma_1}}{\sum_{i=1}^{N} D_i^{-\gamma_j}} \qquad (1)$$

R_0 – distance between target pair (receiver and transmitter)
D_i – distances between target receiver and interfering transmitters
g – transmit power (assumed to be constant for all the transmitters)
γ_j – propagation exponents (may vary from 2 to 6 depending on details of propagation environment)
N – number of interfering pairs

Model for SIR analysis

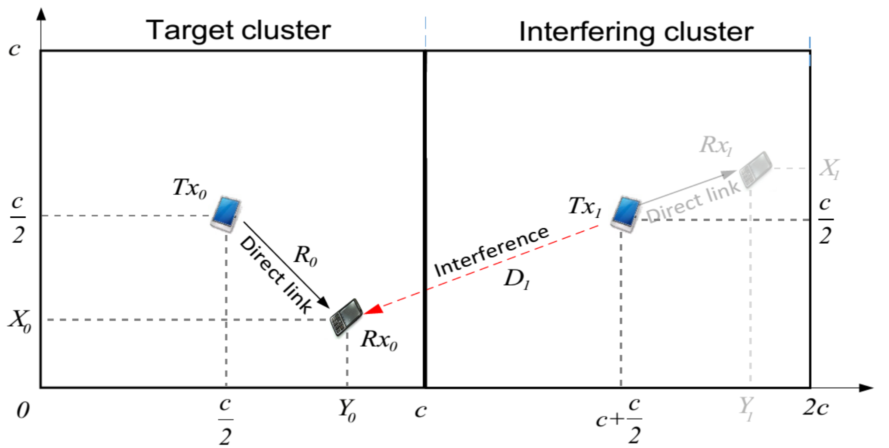


Figure 3.a **Downlink Model**

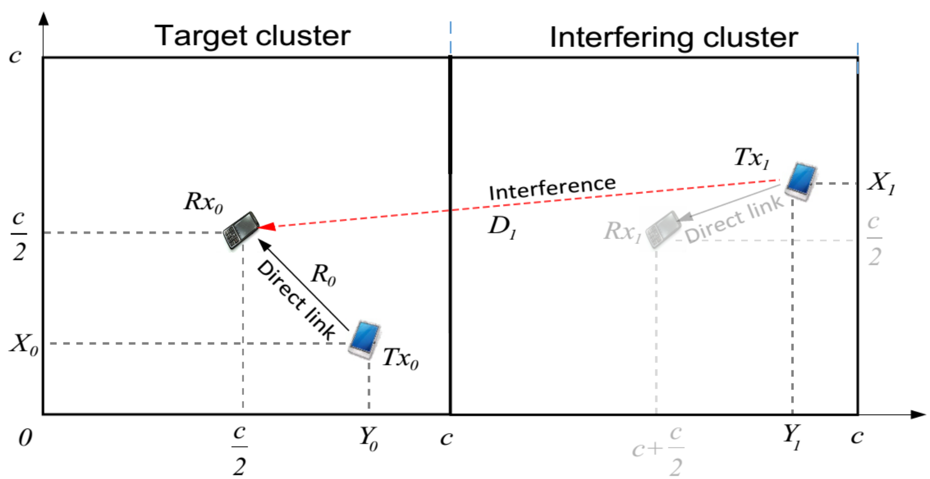


Figure 3.b **Uplink Model**

$$SIR = \frac{R_0^{-\gamma_1}}{D_1^{-\gamma_2}} \qquad (2)$$

$R_0 = R_0(X_0, Y_0)$ – distance between target receiver and transmitter

D_1 – distance between target receiver and interfering transmitter

$D_1 = D_1(X_0, Y_0)$ – Downlink

$D_1 = D_1(X_0, Y_0, X_1, Y_1)$ – Uplink

Rx_0 – Target receiver coordinates

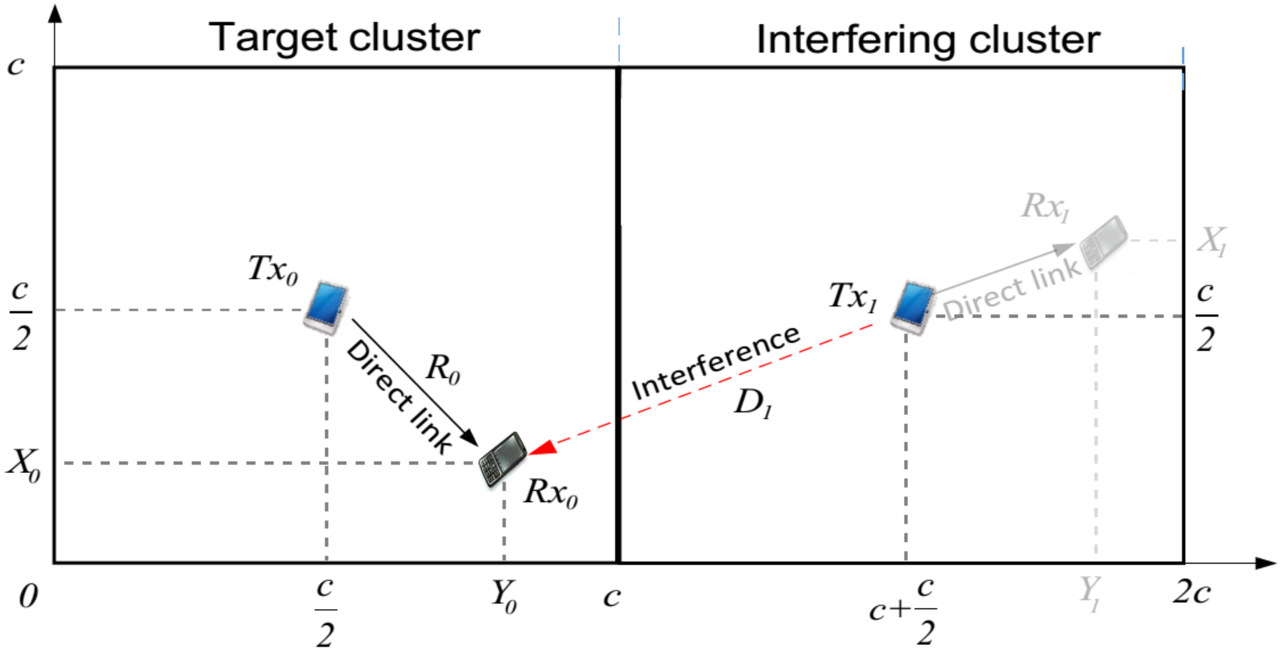
Rx_1 – Coordinates of receiver from interfering cell

Tx_0 – Target transmitter coordinates

Tx_1 – Interfering transmitter coordinates

$a = b = c$ – clusters sides length

Downlink model (1/2)



$$S(x) = \int_0^{\infty} 1_{\Omega}(x, y_2) W(x, y_2)\, dy_2 \qquad (3)$$

$$W(y_1, y_2) = \frac{4}{c^2} \frac{y_1^{\frac{1}{\gamma_1}} \, y_2^{\frac{\gamma_2}{\gamma_1}+1}}{\sqrt{2c^2\left(y_1^{\frac{2}{\gamma_1}} \, y_2^{\frac{2\gamma_2}{\gamma_1}} + y_2^2\right) - (c - y_2^2)^2 - c^4}} \qquad (4)$$

$$\Omega = \Omega_1 \cup \Omega_2 \cup \Omega_3 \cup \Omega_4$$

Downlink model (2/2)

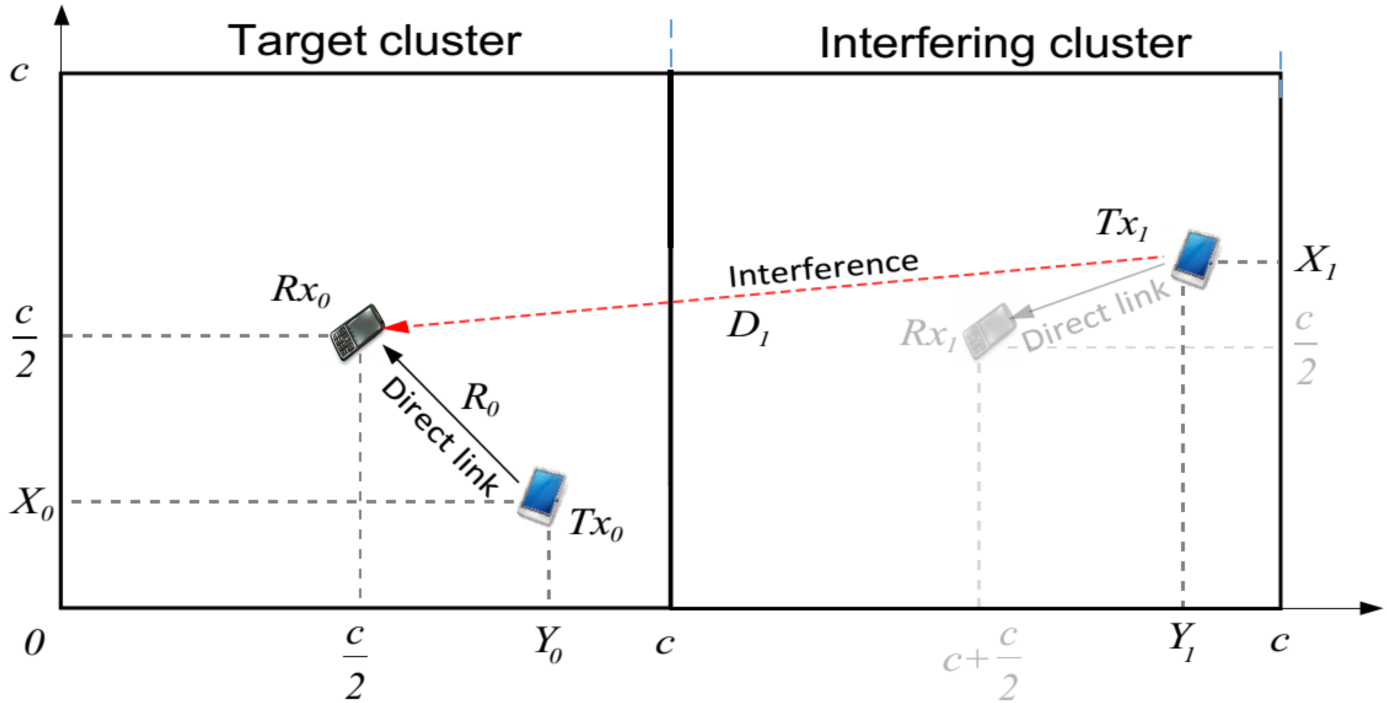
$$\Omega_1=\left\{(y_1,y_2):\frac{c}{2}\le y_1^{\frac{1}{\gamma_1}}y_2^{\frac{\gamma_2}{\gamma_1}}\le\frac{c}{\sqrt{2}},\ c-y_1^{\frac{1}{\gamma_1}}y_2^{\frac{\gamma_2}{\gamma_1}}\le y_2\le y_1^{\frac{1}{\gamma_1}}y_2^{\frac{\gamma_2}{\gamma_1}}\right\},$$

$$\Omega_2=\left\{(y_1,y_2):\frac{c}{\sqrt{2}}\le y_1^{\frac{1}{\gamma_1}}y_2^{\frac{\gamma_2}{\gamma_1}}\le c,\ c-y_1^{\frac{1}{\gamma_1}}y_2^{\frac{\gamma_2}{\gamma_1}}\le y_2\le\sqrt{c^2+y_1^{\frac{2}{\gamma_1}}y_2^{\frac{2\gamma_2}{\gamma_1}}-\sqrt{4c^2y_1^{\frac{2}{\gamma_1}}y_2^{\frac{2\gamma_2}{\gamma_1}}-c^4}}\right\},$$

$$\Omega_3=\left\{(y_1,y_2):c\le y_1^{\frac{1}{\gamma_1}}y_2^{\frac{\gamma_2}{\gamma_1}}\le\frac{3c}{2},\ y_1^{\frac{1}{\gamma_1}}y_2^{\frac{\gamma_2}{\gamma_1}}-c\le y_2\le\sqrt{c^2+y_1^{\frac{2}{\gamma_1}}y_2^{\frac{2\gamma_2}{\gamma_1}}-\sqrt{4c^2y_1^{\frac{2}{\gamma_1}}y_2^{\frac{2\gamma_2}{\gamma_1}}-c^4}}\right\}, \tag{5}$$

$$\Omega_4=\left\{(y_1,y_2):\frac{3c}{2}\le y_1^{\frac{1}{\gamma_1}}y_2^{\frac{\gamma_2}{\gamma_1}}\le\sqrt{\frac{5}{2}}c,\ \sqrt{y_1^{\frac{2}{\gamma_1}}y_2^{\frac{2\gamma_2}{\gamma_1}}-2c^2}\le y_2\le\sqrt{c^2+y_1^{\frac{2}{\gamma_1}}y_2^{\frac{2\gamma_2}{\gamma_1}}-\sqrt{4c^2y_1^{\frac{2}{\gamma_1}}y_2^{\frac{2\gamma_2}{\gamma_1}}-c^4}}\right\}.$$

Uplink model (1/3)



$$S(x) = \int_0^{\infty} \sum_{i=1}^{6} 1_{\Omega_i}(x, y_2) W_i(x, y_2)\, dy_2 \qquad (7)$$

Uplink model (2/3)

$$W_1(y_1,y_2)=\frac{8y_2^{\frac{2}{\gamma_1}}}{\gamma_1\gamma_2 c^4}(y_1y_2)^{\frac{2}{\gamma_2}-1}\arcsin\left[\frac{\sqrt{-c^2+4(y_1y_2)^{\frac{2}{\gamma_2}}}}{2(y_1y_2)^{\frac{1}{\gamma_2}}}\right]\left(\arcsin\left[\frac{c}{2y_2^{\frac{1}{\gamma_1}}}\right]-\arcsin\left[\frac{\sqrt{-c^2+4y_2^{2/\gamma_1}}}{2y_2^{\frac{1}{\gamma_1}}}\right]\right),(y_1y_2)\in\Omega_1$$

$$W_2(y_1,y_2)=\frac{4\pi y_2^{\frac{2}{\gamma_1}}}{\gamma_1\gamma_2 c^4}\arcsin\left[\frac{\sqrt{-c^2+4(y_1y_2)^{\frac{2}{\gamma_2}}}}{2(y_1y_2)^{\frac{1}{\gamma_2}}}\right],(y_1y_2)\in\Omega_2$$

$$W_3(y_1,y_2)=\frac{8y_2^{\frac{2}{\gamma_1}}}{\gamma_1\gamma_2 c^4}\arcsin\left[\frac{c}{2(y_1y_2)^{\frac{1}{\gamma_2}}}\right]\left(\arcsin\left[\frac{c}{2y_2^{\frac{1}{\gamma_1}}}\right]-\arcsin\left[\frac{\sqrt{-c^2+4y_2^{2/\gamma_1}}}{2y_2^{\frac{1}{\gamma_1}}}\right]\right),(y_1y_2)\in\Omega_3 \qquad (8)$$

$$W_4(y_1,y_2)=\frac{4\pi y_2^{\frac{2}{\gamma_1}}}{\gamma_1\gamma_2 c^4}\arcsin\left[\frac{c}{2(y_1y_2)^{\frac{1}{\gamma_2}}}\right],(y_1y_2)\in\Omega_4$$

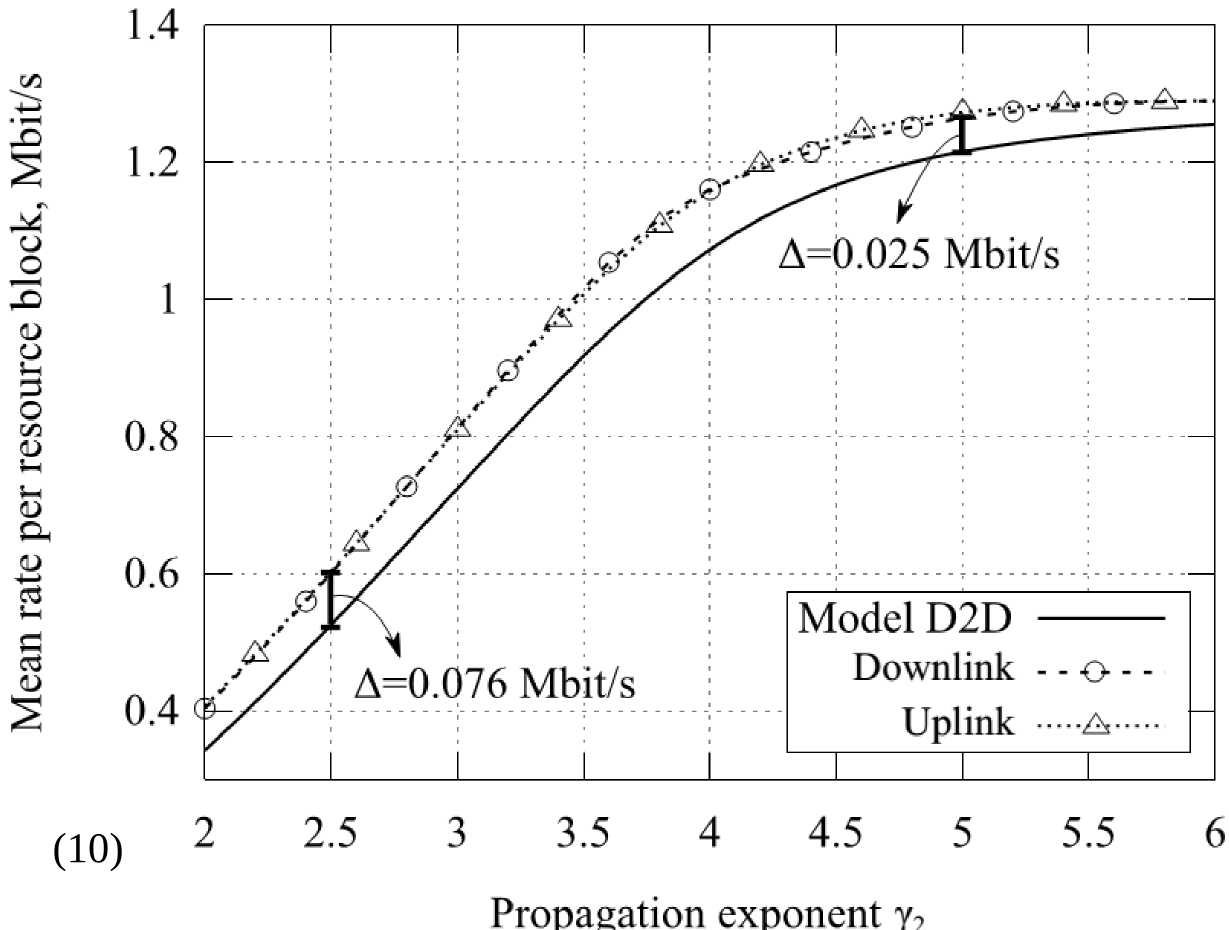
$$W_5(y_1,y_2)=\frac{8y_2^{\frac{2}{\gamma_1}}}{\gamma_1\gamma_2 c^4}\left(\arcsin\left[\frac{c}{2(y_1y_2)^{\frac{1}{\gamma_2}}}\right]-\arcsin\left[\frac{\sqrt{-9c^2+4(y_1y_2)^{\frac{2}{\gamma_2}}}}{2(y_1y_2)^{\frac{1}{\gamma_2}}}\right]\right)\left(\arcsin\left[\frac{c}{2y_2^{\frac{1}{\gamma_1}}}\right]-\arcsin\left[\frac{\sqrt{-c^2+4y_2^{2/\gamma_1}}}{2y_2^{\frac{1}{\gamma_1}}}\right]\right),(y_1y_2)\in\Omega_5$$

$$W_6(y_1,y_2)=\frac{4\pi y_2^{\frac{2}{\gamma_1}}}{\gamma_1\gamma_2 c^4}\left(\arcsin\left[\frac{c}{2(y_1y_2)^{\frac{1}{\gamma_2}}}\right]-\arcsin\left[\frac{\sqrt{-9c^2+4(y_1y_2)^{\frac{2}{\gamma_2}}}}{2(y_1y_2)^{\frac{1}{\gamma_2}}}\right]\right),(y_1y_2)\in\Omega_6$$

Uplink model (3/3)

$$
\begin{aligned}
\Omega_1 &= \left\{ \frac{c^{\gamma_2-\gamma_1}}{\sqrt{2^{2\gamma_2-\gamma_1}}} \le y_1 \le \left(\frac{c}{2}\right)^{\gamma_2-\gamma_1}, \frac{\left(\frac{c}{2}\right)^{\gamma_2}}{y_1} \le y_2 \le \left(\frac{c}{\sqrt{2}}\right)^{\gamma_1} \right\} \cup \left\{ \left(\frac{c}{2}\right)^{\gamma_2-\gamma_1} < y_1 \le \left(\frac{c}{\sqrt{2}}\right)^{\gamma_1-\gamma_2}, \left(\frac{c}{2}\right)^{\gamma_1} \le y_2 \le \left(\frac{c}{\sqrt{2}}\right)^{\gamma_1} \right\} \cup \left\{ \left(\frac{c}{\sqrt{2}}\right)^{\gamma_1-\gamma_2} < y_1 \le \frac{\left(\frac{c}{\sqrt{2}}\right)^{\gamma_2}}{\left(\frac{c}{2}\right)^{\gamma_1}}, \left(\frac{c}{2}\right)^{\gamma_1} \le y_2 \le \frac{\left(\frac{c}{\sqrt{2}}\right)^{\gamma_2}}{\left(\frac{c}{2}\right)^{\gamma_1}} \right\}, \\
\Omega_2 &= \left\{ \left(\frac{c}{2}\right)^{\gamma_2-\gamma_1} \le y_1 \le \frac{\left(\frac{c}{\sqrt{2}}\right)^{\gamma_2}}{\left(\frac{c}{2}\right)^{\gamma_1}}, \frac{\left(\frac{c}{2}\right)^{\gamma_2}}{y_1} \le y_2 \le \left(\frac{c}{2}\right)^{\gamma_1} \right\} \cup \left\{ y_1 > \frac{\left(\frac{c}{\sqrt{2}}\right)^{\gamma_2}}{\left(\frac{c}{2}\right)^{\gamma_1}}, \frac{\left(\frac{c}{2}\right)^{\gamma_2}}{y_1} \le y_2 \le \frac{\left(\frac{c}{\sqrt{2}}\right)^{\gamma_2}}{y_1} \right\}, \\
\Omega_3 &= \left\{ \left(\frac{c}{\sqrt{2}}\right)^{\gamma_1-\gamma_2} \le y_1 \le \frac{\left(\frac{c}{\sqrt{2}}\right)^{\gamma_2}}{\left(\frac{c}{2}\right)^{\gamma_1}}, \frac{\left(\frac{c}{\sqrt{2}}\right)^{\gamma_2}}{y_1} \le y_2 \le \left(\frac{c}{\sqrt{2}}\right)^{\gamma_1} \right\} \cup \left\{ \frac{\left(\frac{c}{\sqrt{2}}\right)^{\gamma_2}}{\left(\frac{c}{2}\right)^{\gamma_1}} < y_1 \le \frac{\frac{c^{\gamma_2-\gamma_1}}{\left(\sqrt{2}\right)^{2\gamma_2-\gamma_1}}}{3^{-\gamma_2}}, \left(\frac{c}{2}\right)^{\gamma_1} \le y_2 \le \left(\frac{c}{\sqrt{2}}\right)^{\gamma_1} \right\} \cup \left\{ \frac{\frac{c^{\gamma_2-\gamma_1}}{\left(\sqrt{2}\right)^{2\gamma_2-\gamma_1}}}{3^{-\gamma_2}} < y_1 \le \frac{\left(\frac{3c}{2}\right)^{\gamma_2}}{\left(\frac{c}{2}\right)^{\gamma_1}}, \left(\frac{c}{2}\right)^{\gamma_1} \le y_2 \le \frac{\left(\frac{3c}{2}\right)^{\gamma_2}}{y_1} \right\}, \\
\Omega_4 &= \left\{ \frac{\left(\frac{c}{\sqrt{2}}\right)^{\gamma_2}}{\left(\frac{c}{2}\right)^{\gamma_1}} \le y_1 \le \frac{\left(\frac{3c}{2}\right)^{\gamma_2}}{\left(\frac{c}{2}\right)^{\gamma_1}}, \frac{\left(\frac{c}{\sqrt{2}}\right)^{\gamma_2}}{y_1} \le y_2 \le \left(\frac{c}{2}\right)^{\gamma_1} \right\} \cup \left\{ y_1 > \frac{\left(\frac{3c}{2}\right)^{\gamma_2}}{\left(\frac{c}{2}\right)^{\gamma_1}}, \frac{\left(\frac{c}{\sqrt{2}}\right)^{\gamma_2}}{y_1} \le y_2 \le \frac{\left(\frac{3c}{2}\right)^{\gamma_2}}{y_1} \right\}, \\
\Omega_5 &= \left\{ \frac{\left(\frac{3c}{2}\right)^{\gamma_2}}{\left(\frac{c}{\sqrt{2}}\right)^{\gamma_1}} \le y_1 \le c^{\gamma_2-\gamma_1}\sqrt{\frac{5^{\gamma_2}}{2^{\gamma_2-\gamma_1}}}, \frac{\left(\frac{3c}{2}\right)^{\gamma_2}}{y_1} \le y_2 \le \left(\frac{c}{\sqrt{2}}\right)^{\gamma_1} \right\} \cup \left\{ c^{\gamma_2-\gamma_1}\sqrt{\frac{5^{\gamma_2}}{2^{\gamma_2-\gamma_1}}} < y_1 \le \frac{\left(\frac{3c}{2}\right)^{\gamma_2}}{\left(\frac{c}{2}\right)^{\gamma_1}}, \frac{\left(\frac{3c}{2}\right)^{\gamma_2}}{y_1} \le y_2 \le \frac{c^{\gamma_2}\left(\sqrt{\frac{5}{2}}\right)^{\gamma_2}}{y_1} \right\} \cup \left\{ \frac{\left(\frac{3c}{2}\right)^{\gamma_2}}{\left(\frac{c}{2}\right)^{\gamma_1}} < y_1 \le \frac{c^{\gamma_2}\left(\sqrt{\frac{5}{2}}\right)^{\gamma_2}}{\left(\frac{c}{2}\right)^{\gamma_1}}, \left(\frac{c}{2}\right)^{\gamma_1} \le y_2 \le \frac{c^{\gamma_2}\left(\sqrt{\frac{5}{2}}\right)^{\gamma_2}}{y_1} \right\}, \\
\Omega_6 &= \left\{ \frac{\left(\frac{3c}{2}\right)^{\gamma_2}}{\left(\frac{c}{2}\right)^{\gamma_1}} \le y_1 \le \frac{c^{\gamma_2}\left(\sqrt{\frac{5}{2}}\right)^{\gamma_2}}{\left(\frac{c}{2}\right)^{\gamma_1}}, \frac{\left(\frac{3c}{2}\right)^{\gamma_2}}{y_1} \le y_2 \le \left(\frac{c}{2}\right)^{\gamma_1} \right\} \cup \left\{ y_1 > \frac{c^{\gamma_2}\left(\sqrt{\frac{5}{2}}\right)^{\gamma_2}}{\left(\frac{c}{2}\right)^{\gamma_1}}, \frac{\left(\frac{3c}{2}\right)^{\gamma_2}}{y_1} \le y_2 \le \frac{c^{\gamma_2}\left(\sqrt{\frac{5}{2}}\right)^{\gamma_2}}{y_1} \right\}.
\end{aligned} \tag{9}
$$

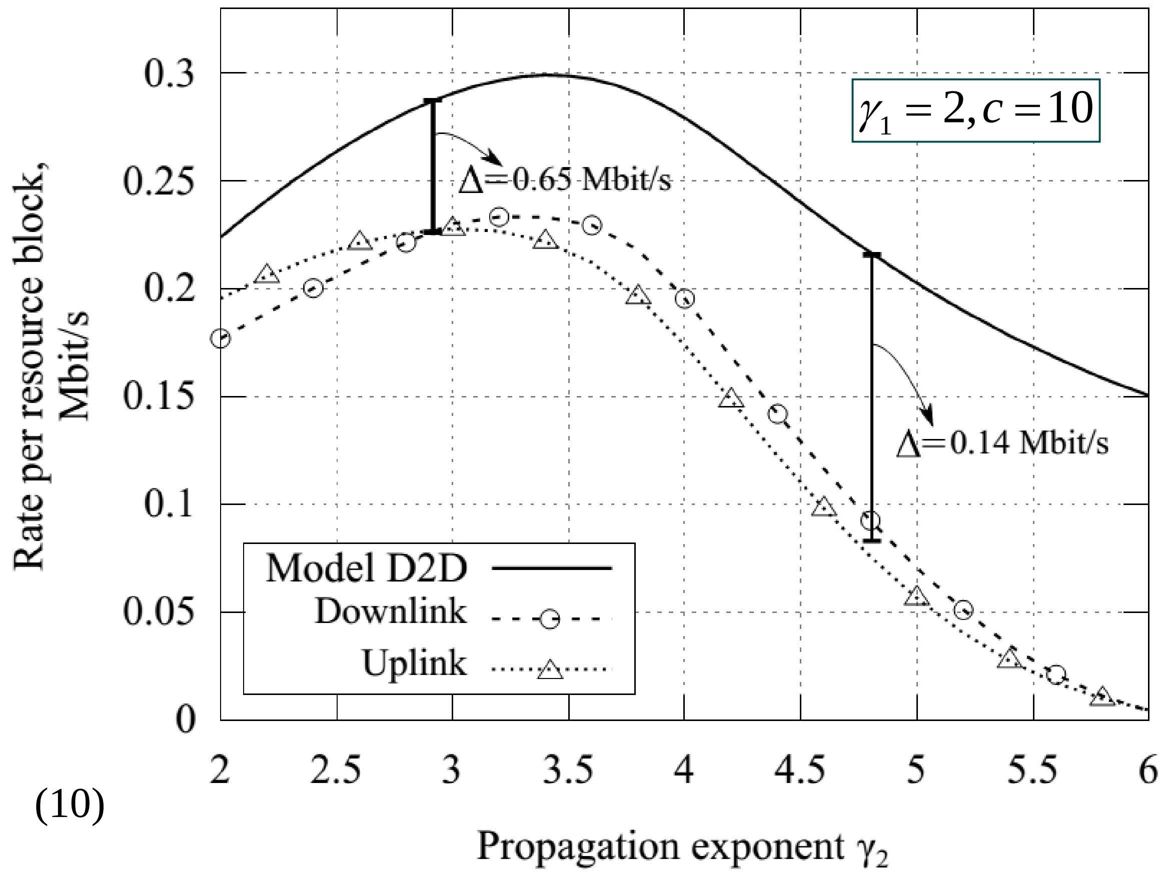
The mean of the achievable D2D rate



$$R = \frac{w}{b}\log_2\left(1 + \frac{SIR}{a}\right) \qquad (10)$$

$$C = S\left[a\left(2^{bx/w} - 1\right)\right]2^{bx/w}\frac{ab\ln 2}{w} \qquad (11)$$

The standard deviation of the achievable D2D rate



$$R = \frac{w}{b}\log_2\left(1 + \frac{SIR}{a}\right) \qquad (10)$$

$$C = S\left[a\left(2^{bx/w} - 1\right)\right]2^{bx/w}\frac{ab\ln 2}{w} \qquad (11)$$

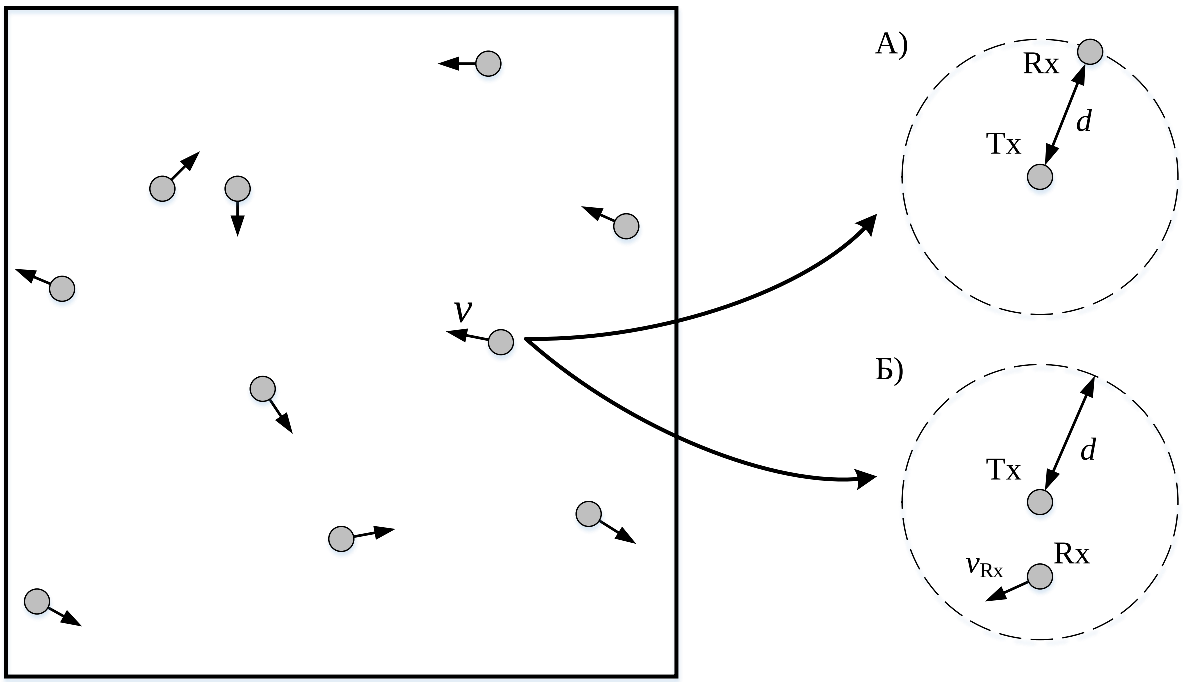
Ongoing Projects: Project 4

- Resource allocation in wireless networks with random resource requirements
- Interruption probability analysis for Licensed Shared Access (LSA) regulatory framework
- Stochastic geometry models and SIR analysis in D2D wireless networks
- **Modelling users' mobility**

Project 4 publications

1. Orlov, Yu. N., S. L. Fedorov, A. K. Samouylov, Yu. V. Gaidamaka, and D. A. Molchanov. 2016. Simulation of devices mobility to estimate wireless channel quality metrics in 5G networks. In: ICNAAM-2016. AIP Conference Proceedings 1863, 090005-1-090005-3. 2017. NY, USA: AIP Publishing. doi: 10.1063/1.4992270.
2. Orlov Yu., Kirina-Lilinskaya E., Samouylov A., Ometov A., Molchanov D., Gaidamaka Yu., Andreev S., Samouylov K. 2017. Time-Dependent SIR Analysis in Shopping Malls Using Fractal-Based Mobility Models. In: Koucheryavy Y. et al. (Eds.): Wired/Wireless Internet Communications. WWIC 2017. LNCS 10372. Springer, Cham. doi: 10.1007/978-3-319-61382-6_2.
3. Fedorov S., Ivchenko A., Orlov Yu., Samuilov A., Ometov A., Moltchanov D., Gaidamaka Yu., Samuilov K.. 2017. Analyzing D2D Mobility: Framework for Steady Communications and Outage Periods Prediction // Proc. of the 40th International Conference on Telecommunications and Signal Processing TSP-2017 (July 5-7, 2017, Barcelona, Spain.). IEEE 2017. doi: 10.1109/TSP.2017.8075945.
4. Ivchenko A., Orlov Y., Samouylov A., Molchanov D., Gaidamaka Yu. 2017. Characterizing Time-Dependent Variance and Coefficient of Variation of SIR in D2D Connectivity. In: Galinina O. et al. (Eds.): NEW2AN 2017, ruSMART 2017, NsCC 2017. LNCS 10531. Springer, Cham. doi: 10.1007/978-3-319-67380-6_49.
5. Gaidamaka Yu., Kirina-Lilinskaya E., Orlov Yu., Samouylov A., Molchanov D. 2017. Construction of the stability indicator for wireless D2D communication in a case of fractal random walk // A. Dudin et al. (Eds.): ITMM 2017, CCIS 800, pp. 324-335, 2017. Springer. doi: 10.1007/978-3-319-68069-9 26.

Modelling users' mobility



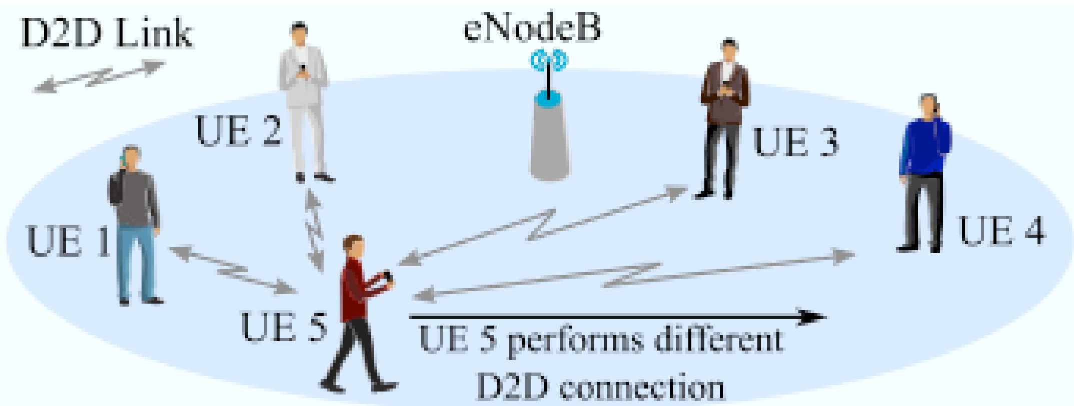
Tx – transmitter

Rx - receiver

d - max radius between receiver and transmitter

v - drift (Brownian motion)

Problem statement: moving devices

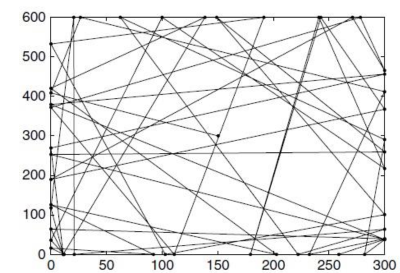


useful signal

interference signal

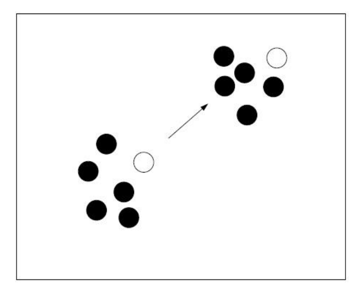
Entity mobility models:

mobile nodes movements are independent of each other

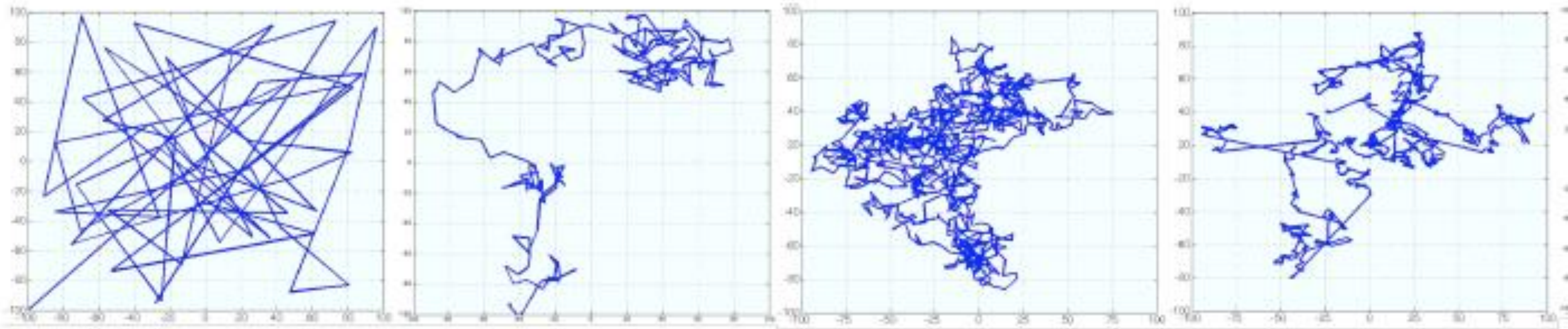


Group mobility models:

mobile nodes movements are dependent on each other



Approach 1: synthetic mobility models for ad hoc network



a) RWP b) RDM c) BM d) LF

a) **Random Waypoint (RWP)**: random directions and speeds, includes pause times between changes in destination and speed

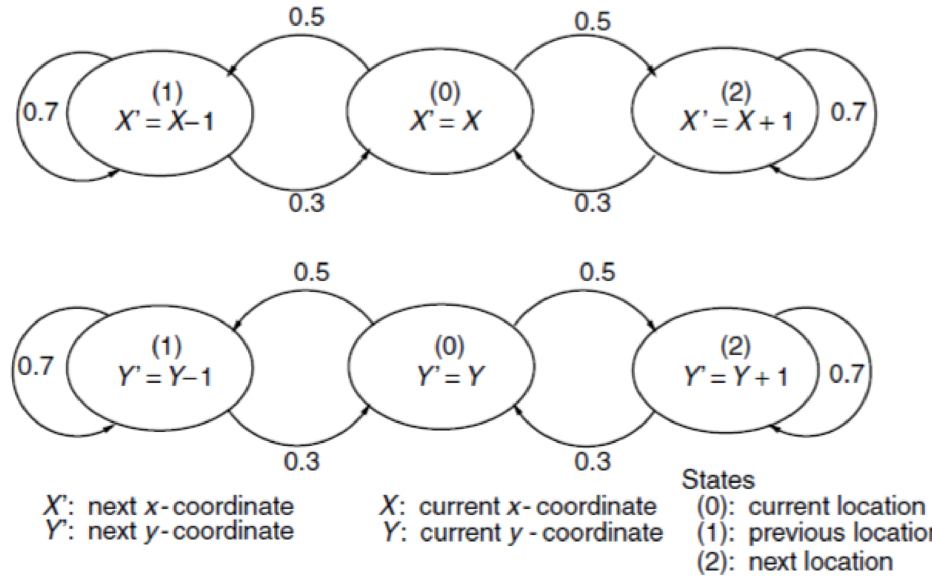
b) **Random Direction (RDM)**: forces MNs to travel to the edge of the simulation area before changing direction and speed

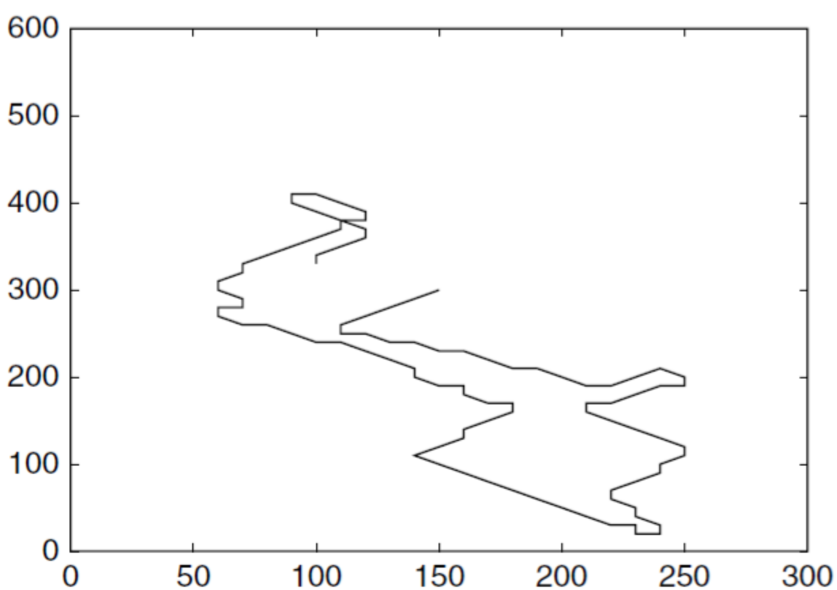
c) **Brownian Motion (BM)**: both the step size and the mobility duration tend to zero, such that their ratio remains constant

d) **Lévy Flight (LF)**: multiple short "runs" interchange with occasional long-distance travels

Approach 1: probabilistic version of Random Walk

e) Random Walk (RW):
random directions and speeds
at each time step
with probability matrix P





$$P = \begin{bmatrix} P(0,0) & P(0,1) & P(0,2) \\ P(1,0) & P(1,1) & P(1,2) \\ P(2,0) & P(2,1) & P(2,2) \end{bmatrix}$$

$$P1 = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.3 & 0.7 & 0 \\ 0.3 & 0 & 0.7 \end{bmatrix}$$

Approach 2:
kinetic approach to random functional analysis

Kinetic equation of Fokker-Planck type:

$$\frac{\partial f}{\partial t}+\frac{\partial}{\partial x}\left(u(x,t)f\right)-\frac{B(t)}{2}\frac{\partial^2 f}{\partial x^2}=0 \quad - \text{ motion equation} \qquad (5)$$

$f\left(x,t\right)$ - coordinates increment distribution function, continuously differentiable function of coordinates and time (distance from moving transmitter to moving receiver)

$u\left(x,t\right)$ - drift velocity

$B(t)$ - diffusion coefficient

Trajectory Modelling Method

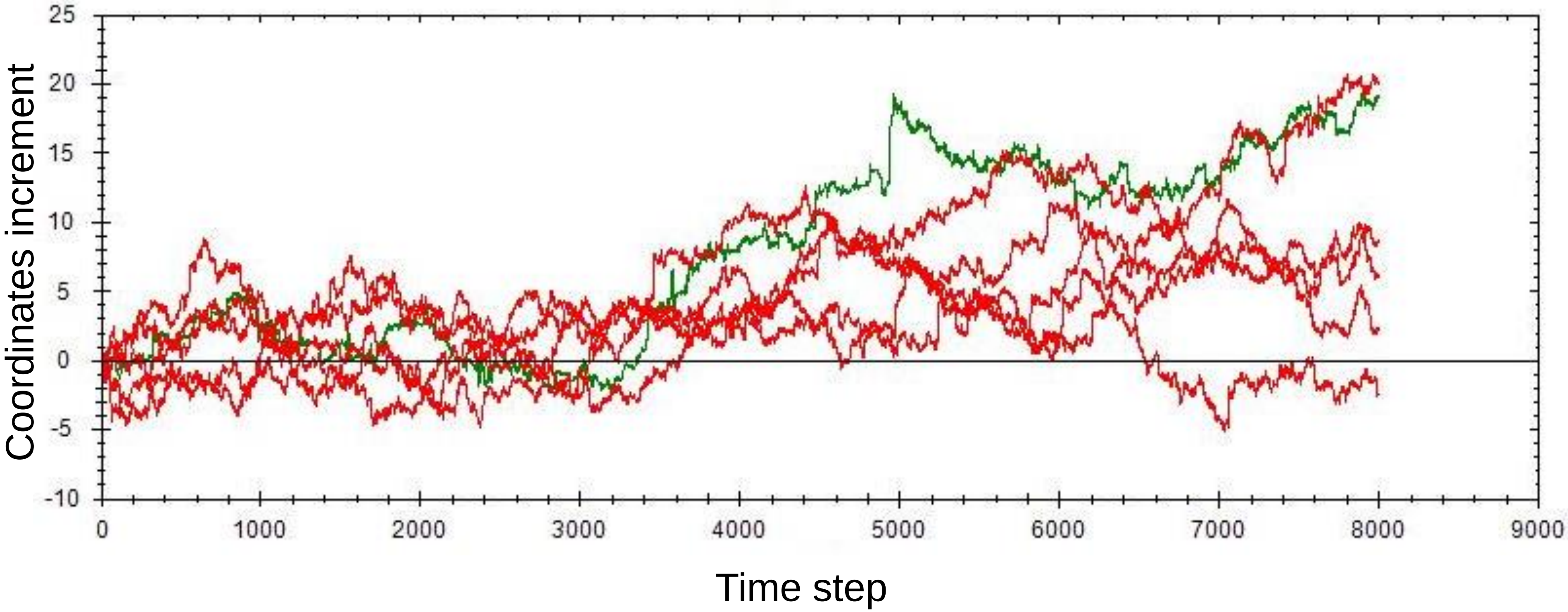
At time step $k = 1, 2, ..., T$ $\quad y_k \in Uni\left[0,1\right] \quad \rightarrow \quad$ (5)

$y_k = F\left(x_k, k\right) \quad \Rightarrow \quad x_k = F^{-1}\left(y_k, k\right)$ - inversion of

$$F(x,t) = \left(nx - j\right) \cdot f_{j+1}(t) + \sum_{k=1}^{j} f_k(t), \;\; x \in [(j-1)/n;\; j/n], \quad j = 1 \div n. \qquad (6)$$

At each time step we repeat the procedure for each of *N* devices.
So after *T* steps we draw *N* trajectories of length *T*.

Trajectories Simulation



Signal-to-Interference Ratio (SIR) as a non-local functional

We have N random trajectories $\vec{r}_i(t_k)$, $i=1,2,\ldots,N$,
for any time step $t = 1, 2, \ldots, T$.

Let us construct the SIR value for the trajectories i=1 and j=2:

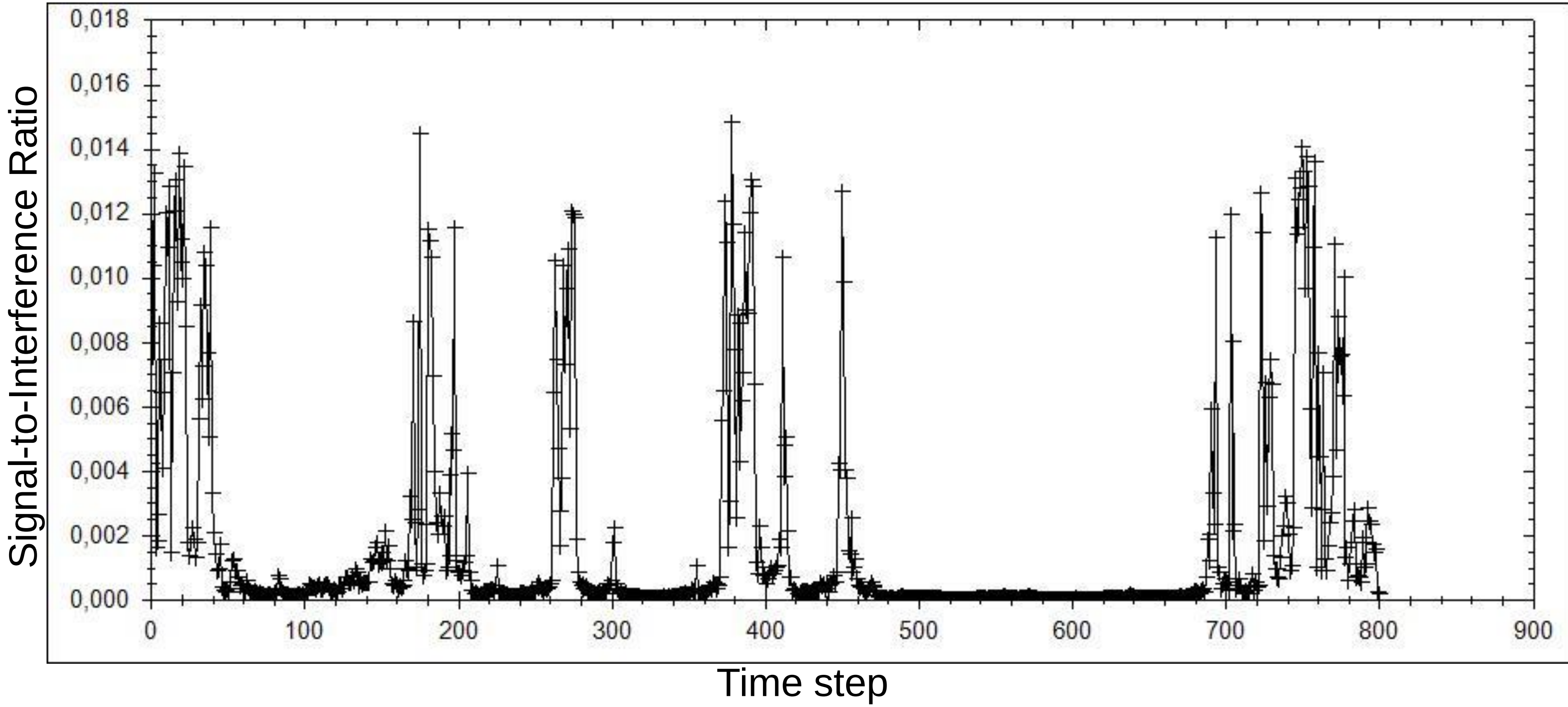
$$S\left(\vec{r}_1(t),\vec{r}_2(t)\right)=\frac{\varphi_{12}}{\sum_{j=3}^{N}\varphi_{1j}},\quad \varphi_{ij}=\varphi\left(\left|\vec{r}_i(t)-\vec{r}_j(t)\right|\right)=\frac{1}{\left|\vec{r}_i(t)-\vec{r}_j(t)\right|^2}$$

SIR approximation with accuracy $o(1/N)$:

$$S(t)\equiv S(r(t))=\frac{\phi(r(t))}{NU(r(t))}, \qquad (7)$$

where $U(r,t)=\int_V \phi\left(\left|\vec{r}-\vec{r}'\right|\right) f\left(\vec{r}',t\right)d\vec{r}'; \quad \vec{r}=\vec{r}_1(t)-\vec{r}_2(t)$

The SIR Simulation



SIR average evolution equation

$$N\frac{dq}{dt}=\int_{-\infty}^{\infty}\left(\frac{\varphi(x)}{U(x,t)}\frac{\partial f(x,t)}{\partial t}-\frac{\varphi(x)}{U^2(x,t)}\frac{\partial U(x,t)}{\partial t}f(x,t)\right)dx \qquad (8)$$

$$N\frac{dq}{dt}=\int_{-\infty}^{\infty}\left(u(x,t)f(x,t)-\frac{B(t)}{Z(\alpha)}G(x,t)\right)\cdot\frac{\partial}{\partial x}\left(\frac{\varphi(x)}{U(x,t)}\right)dx+$$

$$+\int_{-\infty}^{\infty}\frac{\varphi(x)f(x,t)}{U^2(x,t)}\frac{\partial}{\partial x}\left(\int_{-\infty}^{\infty}\varphi(|x-y|)\left(u(y,t)f(y,t)-\frac{B(t)}{Z(\alpha)}G(y,t)\right)dy\right)dx\,.$$

SIR dispersion evolution equation

$$\frac{d\sigma^2(t)}{dt} = -2q(t)\frac{dq(t)}{dt} + 2\int S(x,t)\frac{\partial S(x,t)}{\partial t} f(x,t)dx + \int S^2(x,t)\frac{\partial f(x,t)}{\partial t}dx.$$

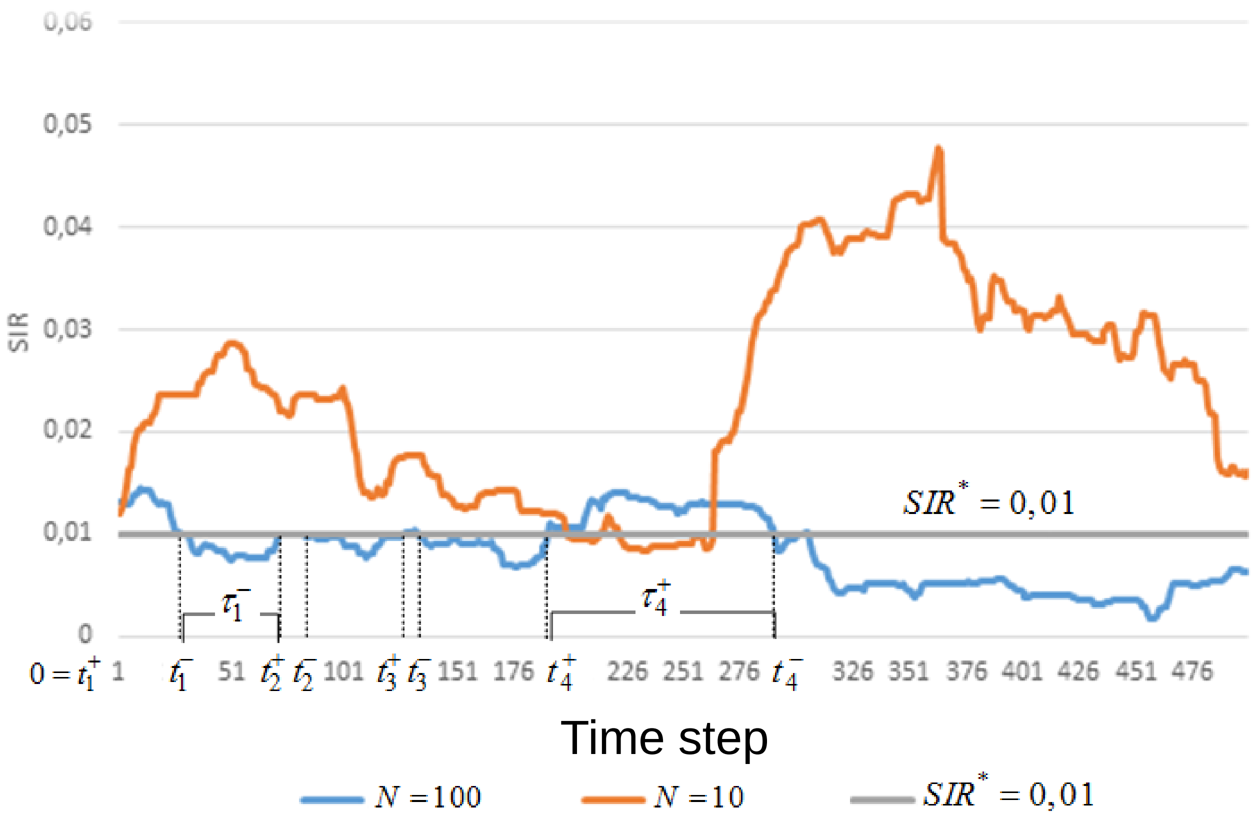
$$2\int S(x,t)\frac{\partial S(x,t)}{\partial t} f(x,t)dx =$$

$$= \frac{2}{N^2}\int \frac{\varphi^2(x)}{U^3(x,t)}\left(\frac{\partial}{\partial x}\int_{-\infty}^{\infty}\varphi(|x-y|)\left(u(y,t)f(y,t) - \frac{B(t)}{Z(\alpha)}G(y,t)\right)dy\right)f(x,t)dx.$$

$$\int S^2(x,t)\frac{\partial f(x,t)}{\partial t}dx = 2\int_{-\infty}^{\infty} S(x,t)\frac{\partial S(x,t)}{\partial x}\left(u(x,t)f(x,t) - \frac{B(t)}{Z(\alpha)}G(x,t)\right)dx =$$

$$= \frac{2}{N^2}\int_{-\infty}^{\infty}\left(\frac{\varphi(x)}{U^2(x,t)}\frac{d\varphi(x)}{dx} - \frac{\varphi^2(x)}{U^3(x,t)}\frac{\partial U(x,t)}{\partial x}\right)\left(u(x,t)f(x,t) - \frac{B(t)}{Z(\alpha)}G(x,t)\right)dx.$$

Average SIR



Parameter	Value
Location area	50 x 50 sq.m
Number of devices	10, 50, 100
Average drift velocity, v	1,3,5,10,40 m/s
Diffusion coefficient, *B*	2
Propagation exponent, γ	3
max radius receiver - transmitter	5 m
SIR threshold, *SIR**	0.01

Goal: CDF of periods of stable D2D connection

Motion Equation

:

$$\frac{\partial f(x,t)}{\partial t}+\frac{\partial\left(u(x,t)\,f(x,t)\right)}{\partial x}=B(t)\frac{\partial^{2\alpha} f(x,t)}{\partial x^{2\alpha}} \qquad (9)$$

$f(x,t)$ - coordinates increment distribution function

$u(x,t)$ - drift velocity

$B(t)$ - diffusion coefficient

α - order of fractional derivative, $0<\alpha<1$

Fractal walk

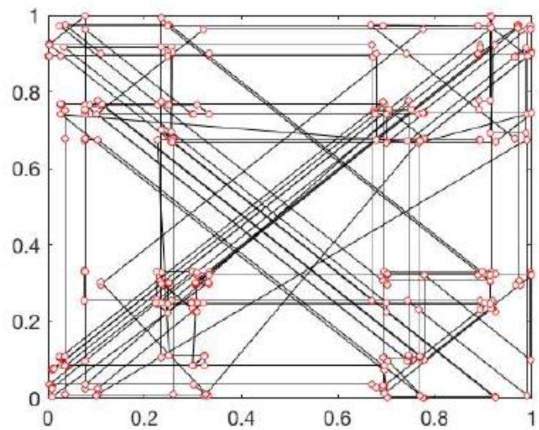


Fig. 1: Sample trajectories on 2D Cantor set.

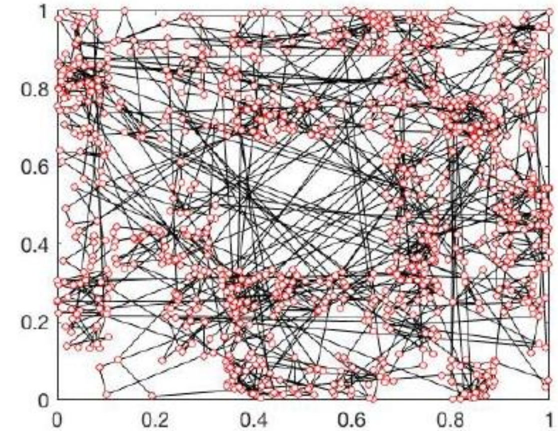


Fig. 2: Sample trajectory on Sierpinski square.

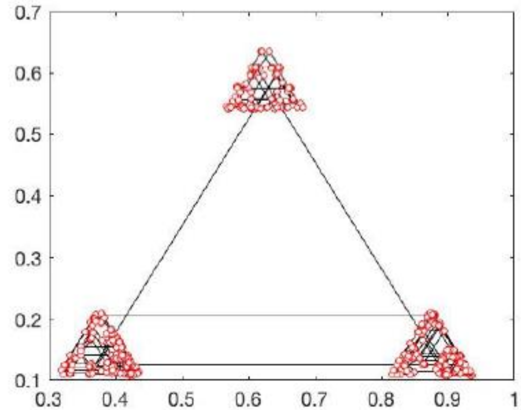


Fig. 3: Sample trajectory on Sierpinski triangle.

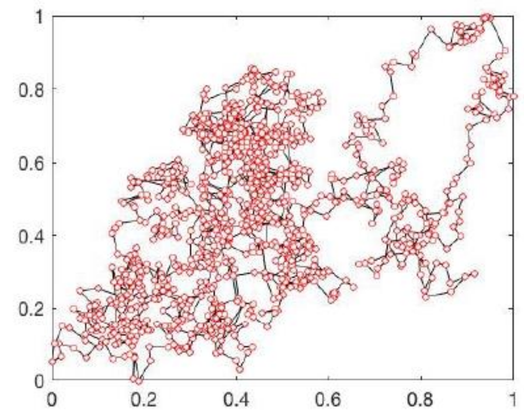


Fig. 4: Sample trajectory of Brownian motion.

SIR characteristics

Average SIR: $$q(t) = \int S(\mathbf{r},t) f(\mathbf{r},t) d\mathbf{r} \qquad (10)$$

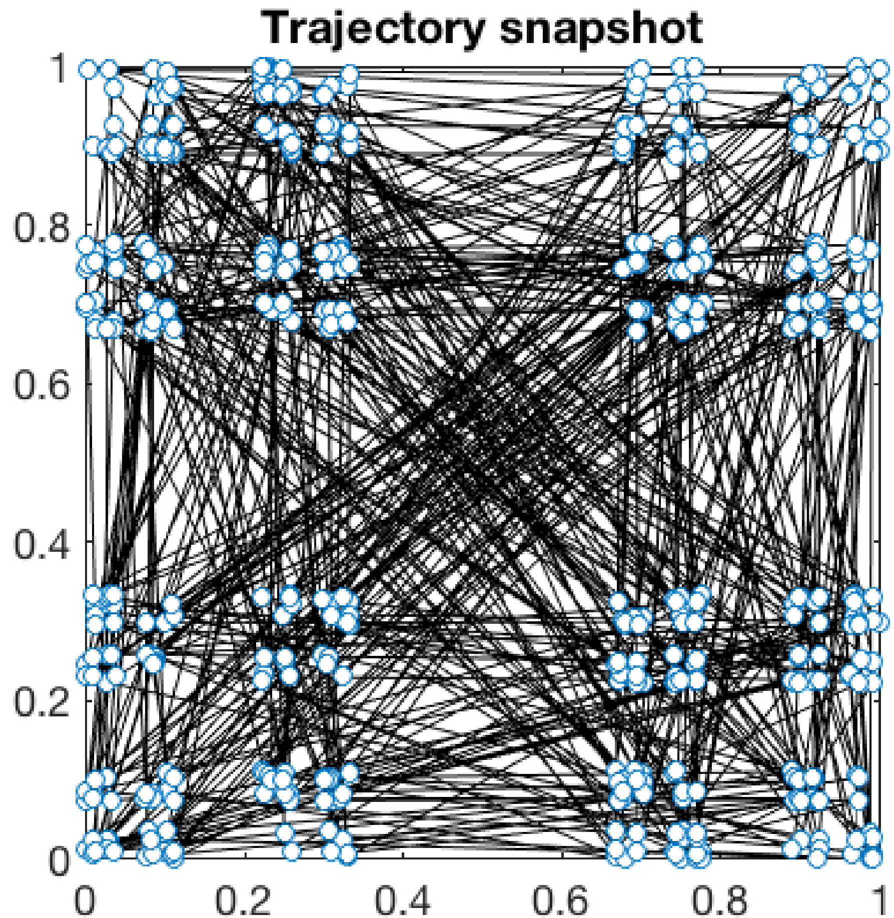
Dispersion of SIR: $$\sigma^2(t) = \int \left(S(\mathbf{r},t) - q(t)\right)^2 f(\mathbf{r},t) d\mathbf{r} \qquad (11)$$

Quality indicator of SIR:

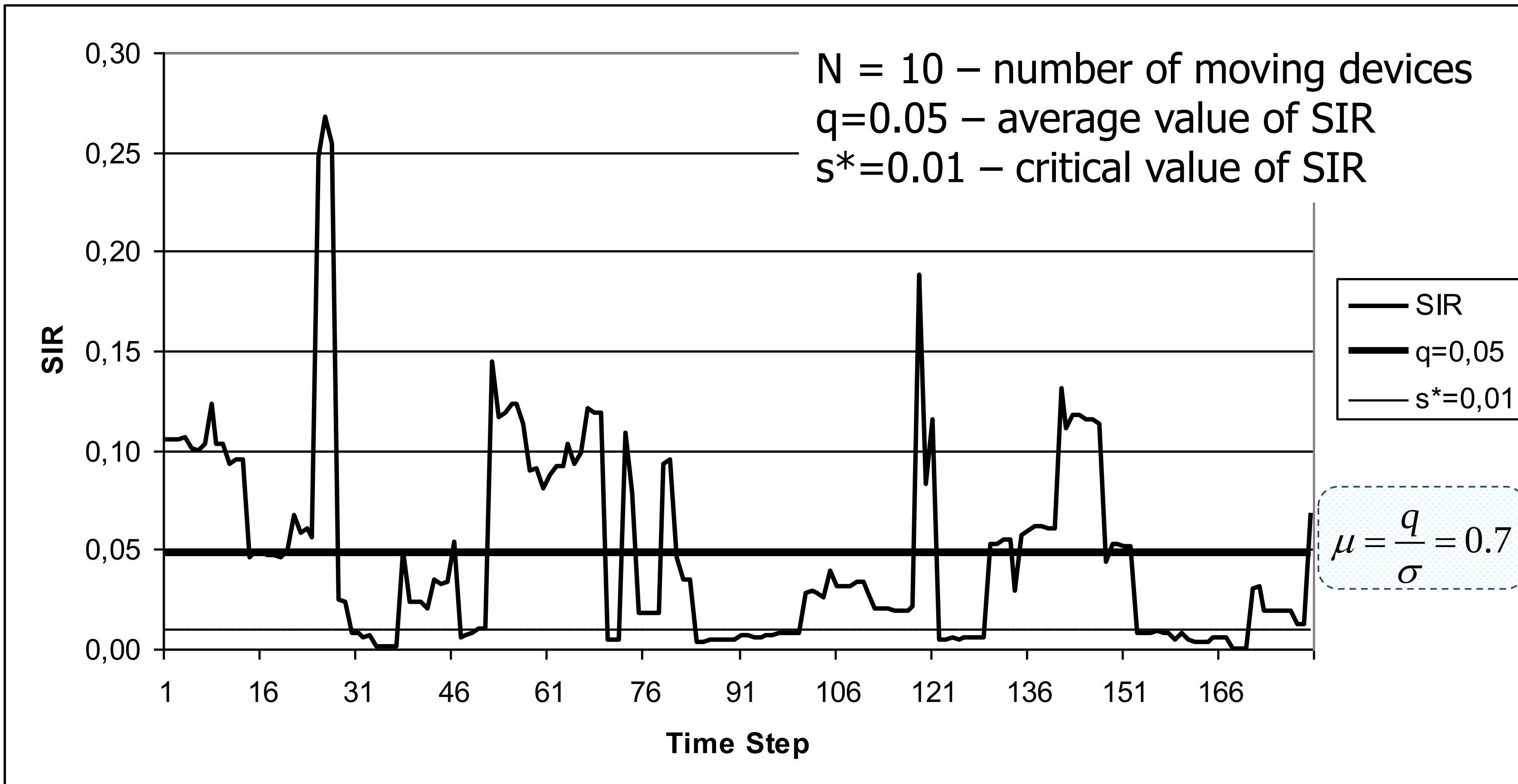
$$\boxed{\mu(t) = \frac{q(t)}{\sigma(t)}} \qquad (12)$$

Fractal walk on a two-dimensional ternary Cantor set

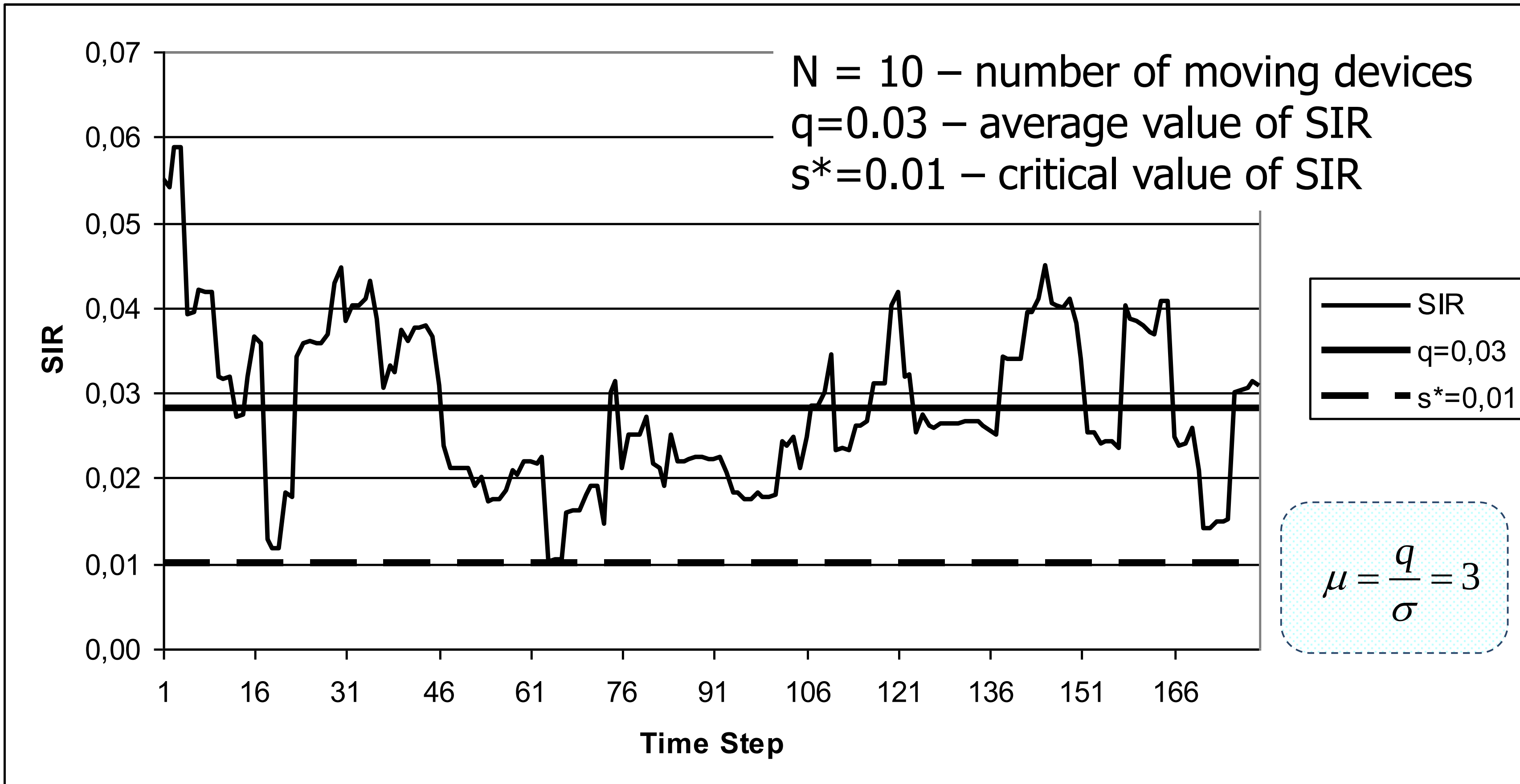
N = 10 trajectories
of points on the ternary
Cantor set



SIR trajectories on Cantor set ($2\alpha = 1,262$): unstable connection



SIR trajectory on Menger set ($2\alpha = 1{,}893$): stable connection



$$\mu = \frac{q}{\sigma} = 3$$

Selected publications (1/2)

1. Petrov V., Samuylov A., Begishev V., Molchanov D., Andreev S., Samouylov K., Koucheryavy E. Vehicle-Based Relay Assistance for Opportunistic Crowdsensing over Narrowband IoT (NB-IoT) // IEEE Internet of Things Journal. – Vol. PP, Issue 99. - p.1. – 2017. doi: 10.1109/JIOT.2017.2670363.

2. Naumov V., Samouylov K. Analysis of multi-resource loss system with state dependent arrival and service rates // Probability in the Engineering and Informational Sciences (Cambridge University Press). – Vol. 31, Issue 4 (G-Networks and their Applications), October 2017, pp. 413-419. doi:10.1017/S0269964817000079.

3. Markova E., Gudkova I., Ometov A., Dzantiev I., Andreev S., Koucheryavy E., Samouylov K. Flexible Spectrum Management in a Smart City within Licensed Shared Access Framework // IEEE Access. – 2017. doi: 10.1109/ACCESS.2017.2758840.

4. Ometov A., Sopin E., Gudkova I., Andreev S., Gaidamaka Yu., Koucheryavy E. Modeling Unreliable Operation of mmWave-based Data Sessions in Mission-Critical PPDR Services // IEEE Access. – October, 2017. doi: 10.1109/ACCESS.2017.2756690.

5. Samuylov A., Ometov A., Begishev A., Kovalchukov R., Moltchanov D., Gaidamaka Yu., Samouylov K., Andreev S., Koucheryavy E. Analytical performance estimation of network-assisted D2D communications in urban scenarios with rectangular cells // Transactions on Emerging Telecommunications Technologies. Volume 28, Issue 2, 1 February 2017, N-e2999.

6. Orlov Yu., Fedorov S., Samuylov A., Gaidamaka Yu., Molchanov D. Simulation of devices mobility to estimate wireless channel quality metrics in 5G networks // AIP Conference Proceedings 1863, 090005 (2017); doi: http://dx.doi.org/10.1063/1.4992270.

Selected publications (2/2)

7. Masek P., Mokrov E., Zeman K., Ponomarenko-Timofeev A., Pyattaev A., Hosek J., Andreev S., Koucheryavy Y., Samouylov K. A Practical Perspective on Highly Dynamic Spectrum Management with LSA // submitted to IEEE Access, 2017.

8. Moltchanov D., Kustarev P., Koucheryavy Y. Analytical modeling and analysis of interleaving on correlated wireless channels // Computer Communications, submitted September 4, 2017.

9. Ometov A., Kozyrev D., Rykov V., Andreev S., Gaidamaka Yu., Koucheryavy Y. Reliability-Centric Analysis of Offloaded Computation in Cooperative Wearable Applications // Wireless Communications and Mobile Computing, accepted, 2017.

10. Kovalchukov R., Samuylov A., Moltchanov D., Ometov A., Andreev S., Koucheryavy Y., Samouylov K. Modeling Three-Dimensional Interference and SIR in Highly Directional mmWave Communications // Proc. of the IEEE Global Communications Conference (GLOBECOM 2017), Singapore, December 4-8, 2017.

11. Mokrov E., Ponomarenko-Timofeev A., Gudkova I., Masek P., Hosek J., Andreev S., Koucheryavy Y., Gaidamaka Yu. Modeling Transmit Power Reduction for a Typical Cell with Licensed Shared Access Capabilities // IEEE Transactions on Vehicular Technology, submitted, 2017.

12. Sopin E., Samouylov K., Vikhrova O., Kovalchukov R., Moltchanov D., Samuylov A. Evaluating a case of downlink uplink decoupling using queuing system with random requirements // Lecture Notes in Computer Science. - 2016. - Vol. 9870. - P. 440-450. https://doi.org/10.1007/978-3-319-46301-8_37

Future projects

6G trend:

to integrate terrestrial wireless with satellite systems for ubiquitous always-on broadband global network coverage.

6G Applications and Technology:

- Smart Homes, Smart Building and Smart Cities
- robotic and autonomous drone delivery
- autonomous transport systems
- Full Immersive Experience

Our projects:

- Interference in Ultra-dense cell networks
- Millimetre Waves for user access
- Modelling cells’ mobility, including group mobility models

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Loss Systems with Random Resource Requirements

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Valeriy Naumov
Service Innovation Research Institute, Helsinki

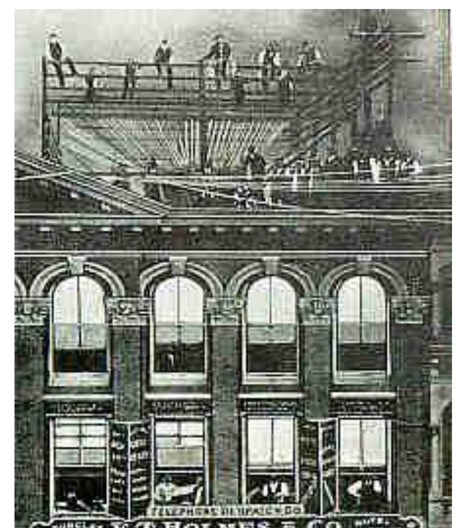
Content

- Loss Systems
- Loss Networks
- Loss Systems with Random Resource Demands
- Loss Systems with Negative Resource Demands
- Loss Networks with Random Resource Demands

Start of the Telecommunications Revolution

- The First Telephone and start of the Telecommunications Revolution – **1876**

Alexander Graham Bell: "Mr. Watson come here – I want to see you"



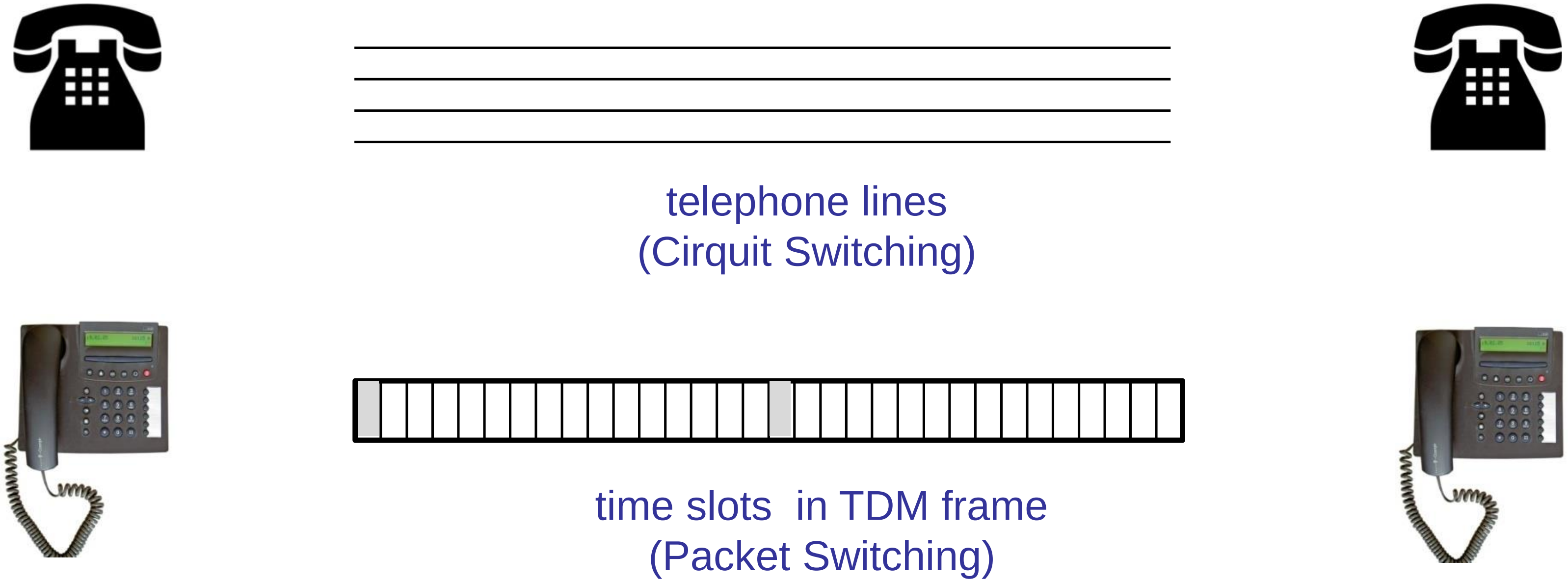
- The First Telephone Exchange – **1877**
- Agner Krarup Erlang was born in **1878**

Agner Krarup Erlang

- Agner Krarup Erlang (1878 – 1929) - the first person studying problems arising in the context of telephone calls.
  - The first paper on these problems – "*The theory of probability and telephone conversations*" (1909).
  - The most important work - "*Solution of some Problems in the Theory of Probabilities of Significance in Automatic Telephone Exchanges*" (1917).
- It is known that a researcher from the Bell Telephone Laboratories learned Danish in order to be able to read Erlang's papers in the original language.

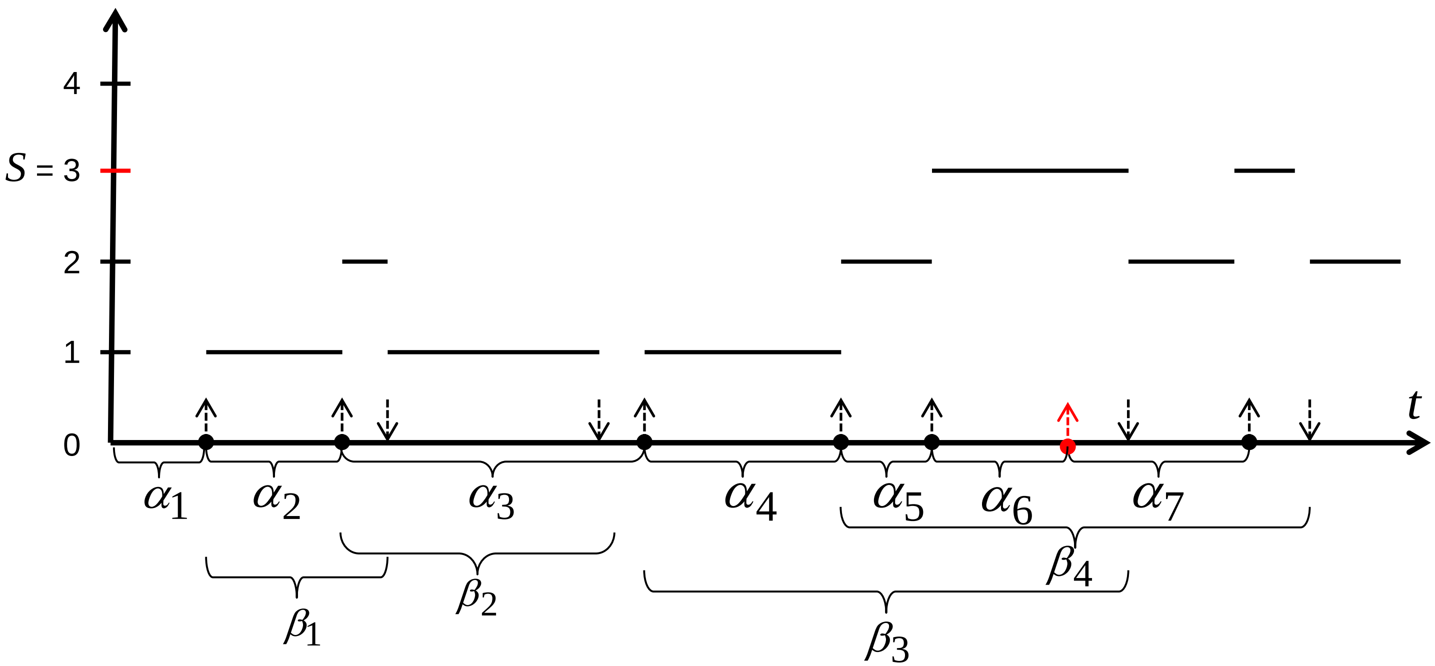
Telephone Trunks

- Trunking is a method for a system to provide network access to many customers by sharing a set of lines



- By studying a telephone trunk in 1917 Erlang worked out a formula, now known as *Erlang's Loss Formula*.

Random process



Telephone Trunk Modelling

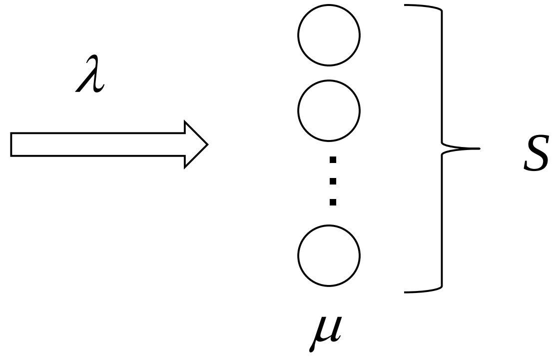
- S servers. Each of them is available if it is not busy;
- Arrival process is the Poisson with the rate λ, i.e. interarrival times α_i are independent and have exponential probability distribution with the mean $1/\lambda$,

$$P(\alpha_i > x) = \exp(-\lambda x), \quad x \geq 0, \quad i = 1,2,\ldots$$

- Service times β_i are independent and have exponential probability distribution with the mean $1/\mu$,

$$P(\beta_i > x) = \exp(-\mu x), \quad x \geq 0, \quad i = 1,2,\ldots$$

- Arriving customer is lost if all servers are busy.

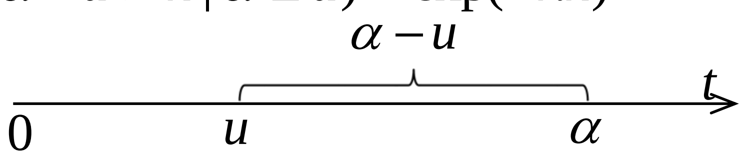


Properties of exponential probability distribution

- Exponential distribution is memoryless, i.e.

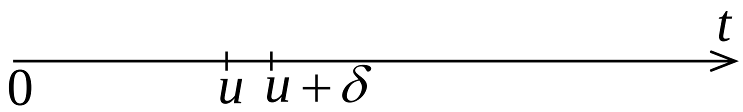
  if $P(\alpha > x) = \exp(-\lambda x)$ for all $x \geq 0$,

  then $P(\alpha - u > x \mid \alpha \geq u) = \exp(-\lambda x)$ for all $u, x \geq 0$.



- If $P(\alpha > x) = \exp(-\lambda x)$ for all $x \geq 0$,

  then $P(u \leq \alpha < u + \delta \mid \alpha \geq u) = \lambda\delta + o(\delta)$, for all $u \geq 0$



- Minimum of exponentially distributed random variables has exponential probability distribution:

  if $P(\alpha_i > x) = \exp(-\lambda_i x),$ for all $x \geq 0,\ i = 1,2,\ldots,n,$

  then $P(\min\{\alpha_i\} > x) = \exp(-\lambda x),\quad x \geq 0,$ where $\lambda = \lambda_1 + \lambda_2 + \ldots + \lambda_n$

State transition diagram

- The set of feasible states is $\mathcal{X} = \{n \in \mathbb{N} \mid 0 \le n \le S\}$

  state 0 – all servers are free,

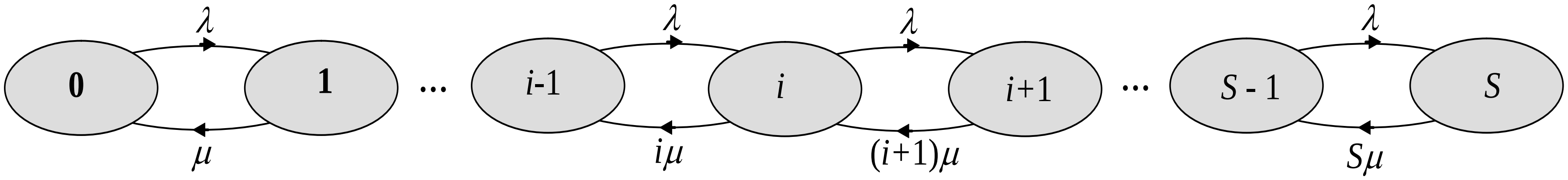
  state 1 – one server is busy, others are free,

  . . .

  state n – n servers are busy and $S - n$ are free,

  . . .

  state S – all servers are busy.



Equilibrium equations

- Transition rates up and down are the same

$$\begin{cases} \lambda p_0 = \mu p_1 \\ \quad \dots \\ \lambda p_{i-1} = i\mu p_i \\ \quad \dots \\ \lambda p_{S-1} = S\mu \ p_S \\ \sum_{i=0}^{S} p_i = 1 \end{cases}$$

- The stationary probability distribution of the number of busy servers is given by

$$\boxed{p_n = \frac{\rho^n}{n!}\left(\sum_{k=0}^{S}\frac{\rho^k}{k!}\right)^{-1}, \qquad n = 0,1,\dots,S,} \qquad (1)$$

where $\rho = \lambda / \mu$ is the mean number of arrivals within the mean service time.

Erlang's Loss Formula

- Call blocking probability: probability that arriving call finds the system busy – propotion of the lost calls
- Time blocking probability: probability that at an arbitrary selected instant of time the system is busy – the proportion of time when the system is busy
- PASTA (Poisson Arrivals See Time Averages) property:

  If arrival process is Poisson then call and time blocking probabilities are the same.
- *Erlang's Loss Formula*: blocking probabilities are given by

$$E_S(\rho) = \frac{\rho^S}{S!}\left(\sum_{k=0}^{S}\frac{\rho^k}{k!}\right)^{-1} \qquad (2)$$

Computation of Loss Formula

- Computation: $E_S(\rho) = \dfrac{1}{C_S(\rho)}$ where

$$C_0(A) = 1 \quad , \qquad C_i(\rho) = 1 + \left(\frac{i}{\rho}\right) C_{i-1}(\rho) \quad , \qquad i = 1,2,\ldots,S$$

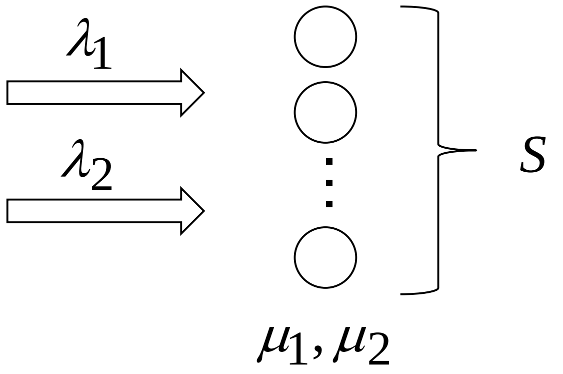
- Integral representation

$$E_S(\rho) = \frac{A^S e^{-\rho}}{\int_\rho^\infty e^{-t} t^S \, dt}$$

- In 1957 Russian mathematician Boris Sevastyanov proved that Erlang Loss Formula remains valid if service times have general distribution.

Two arriving processes

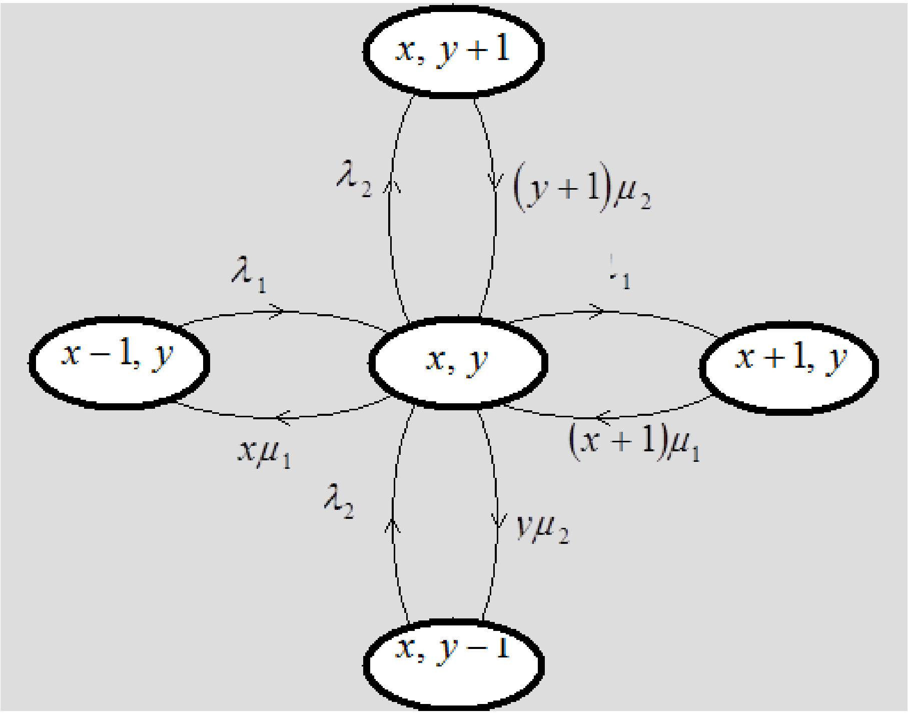
- S servers. Each of them is available if it is not busy;
- Two independent Poisson arrival processes with intensities λ_1 and λ_2
- Service times are independent and have exponential probability distributions with parameters μ_1 and μ_2
- Arriving customer is lost if all servers are busy.



State transition diagram

- The set of feasible states is

$$\mathcal{X} = \{(x, y) \in \mathbb{N}^2 \mid x, y \geq 0,\ x + y \leq S\}$$



Two-dimensional Erlang distribution

- The stationary probability distribution of the number of busy servers is given by

$$\boxed{p_{x,y} = \frac{\dfrac{\rho_1^x}{x!}\,\dfrac{\rho_2^y}{y!}}{\displaystyle\sum_{i=0}^{S}\sum_{j=0}^{S-i}\dfrac{\rho_1^i}{i!}\,\dfrac{\rho_2^j}{j!}}, \qquad x, y \geq 0,\ \ x+y \leq S.} \tag{3}$$

- Blocking probabilities are given by

$$B_1 = B_2 = \frac{\displaystyle\sum_{i=0}^{S}\dfrac{\rho_1^i}{i!}\,\dfrac{\rho_2^{S-i}}{(S-i)!}}{\displaystyle\sum_{i=0}^{S}\sum_{j=0}^{S-i}\dfrac{\rho_1^i}{i!}\,\dfrac{\rho_2^j}{j!}} = E_S(\rho) \tag{4}$$

with $\rho = \rho_1 + \rho_2$.

Multi-dimensional Erlang distribution

- Superposition of independent Poisson processes is the Poisson process with intensity

$$\lambda = \sum_{k=1}^{K} \lambda_k$$

- Probability distribution of the service time is the weighed sum of exponential distributions with the mean:

$$\frac{1}{\mu} = \sum_{k=1}^{K} \frac{\lambda_k}{\lambda}\left(\frac{1}{\mu_k}\right) = \frac{1}{\lambda}\sum_{i=1}^{K}\left(\frac{\lambda_k}{\mu_k}\right) = \frac{1}{\lambda}\sum_{k=1}^{K}\rho_k$$

- Since Erlang Loss Formula remains valid if service times have general distribution, blocking probability is given by $E_S(\rho)$ with

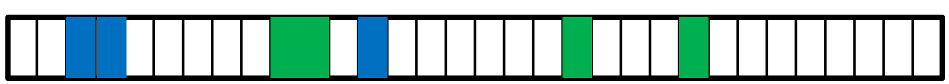
$$\rho = \frac{\lambda}{\mu} = \sum_{k=1}^{K}\rho_k$$

Generalized loss systems

- *Multi-class sources:* Class k customers arrive as a Poisson process with rate λ_k with the mean holding time $1/\mu_k$
- *Simultaneous acquisition of multiple servers:* A class k customer requires to hold s_k servers simultaneously.
- The set of feasible states is

$$\mathcal{X} = \{\mathbf{n} \in \mathbb{N}^K \mid \sum_{k=1}^{K} s_k n_k \leq S\}$$

- Generalized loss systems are used for the performance analysis of high-speed data transmission, that requires multiple TDM slots


Generalized Loss Systems

- The stationary distribution is given by

$$\boxed{p_{\mathbf{n}} = \frac{1}{G}\prod_{k=1}^{K}\frac{\rho_k^{n_k}}{n_k!}, \quad \mathbf{n}\in\mathcal{X}, \qquad G = \sum_{\mathbf{n}\in\mathcal{X}}\prod_{k=1}^{K}\frac{\rho_k^{n_k}}{n_k!},} \tag{5}$$

where

$$\rho_k = \lambda_k / \mu_k$$

- Blocking probabilities for class *i* customers can be calculated as

$$B_i = 1 - \frac{G_i}{G}, \tag{6}$$

where

$$G_i = \sum_{n_1 s_1 + \ldots + n_K s_K + s_i \le S} \prod_{k=1}^{K} \frac{\rho_k^{n_k}}{n_k!}$$

Loss networks

- Simultaneous acquisition of multiple *servers of different types:* There are S_m servers of type m. A class k customers requires to hold s_{km} servers of type m simultaneously
- The preceding formulas for the stationary distribution are valid. Only the set of feasible states is different

$$\mathcal{X} = \{\mathbf{n} \in \mathbb{N}^K \mid \sum_{k=1}^{K} s_{km} n_k \le S_m, m = 1,2,\ldots M\}$$

- **The loss network provides a general model for a circuit-switched network that carries multi-rate traffic**
- The model is equally applicable to bidirectional flows.
- The reverse traffic for a given pair of nodes may have different bandwidth requirements

Loss systems with random resource demands

- Acquisition of multiple *resources of different types:*
  - There are R_m units of resources of type m, $\mathbf{R} = (R_1, \dots, R_M)$
  - The i-th customer of class k requires to hold $r_{km}(i)$ units of resources of type m.
  - Resource demands $\mathbf{r}_k(i) = (r_{k1}(i), \dots, r_{kM}(i)),\ i = 1, 2, \dots$ of class k customers are nonnegative random vectors with cumulative distribution functions $F_k(\mathbf{x})$
- The set of feasible states is given by

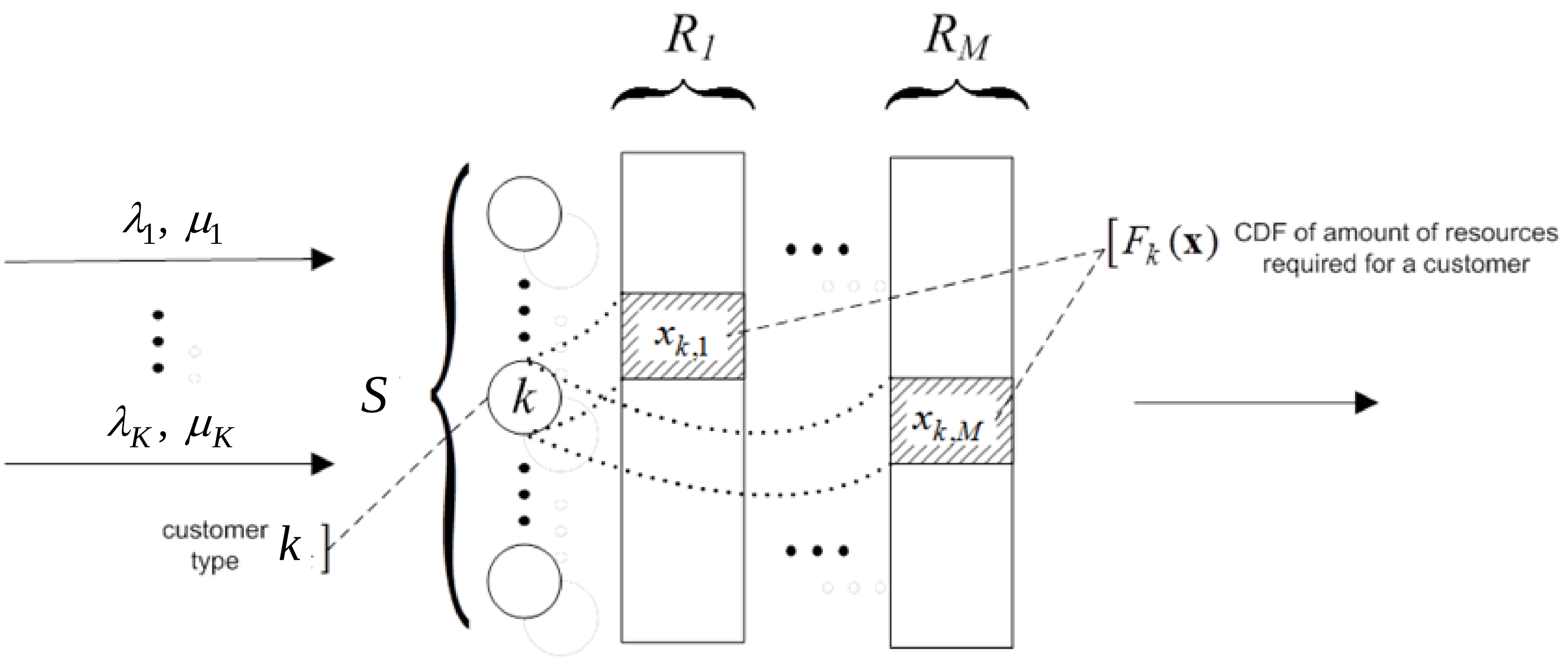
$$\mathcal{X} = \{(\mathbf{n}, \boldsymbol{\gamma}_1, \dots, \boldsymbol{\gamma}_K) \mid \mathbf{n} \in \mathbb{N}^K, \boldsymbol{\gamma}_k \in \mathbb{R}_+^M, k = 1, 2, \dots, K,$$

$$\sum_{k=1}^{K} \boldsymbol{\gamma}_k \le \mathbf{R}, \sum_{k=1}^{K} n_k \le S\}$$

$\mathbf{n} = (n_1, \dots, n_K)$ – population vector

$\boldsymbol{\gamma}_k = (\gamma_{k1}, \dots, \gamma_{kM})$ – vector of resources occupied by class k customers

Loss systems with random resource demands



Loss systems with random resource demands (continued)

- Cumulative distribution functions of the stationary distribution are given by

$$P_{\mathbf{n}}(\mathbf{x}_1,\ldots,\mathbf{x}_K) = \frac{1}{G}\prod_{k=1}^{K}\frac{\rho_k^{n_k}}{n_k!}F_k^{*n_k}(\mathbf{x}_k), \quad (\mathbf{n},\mathbf{x}_1,\ldots,\mathbf{x}_K)\in\mathcal{X},$$

$$G = \sum_{n_1+\ldots+n_K\le S}(F_1^{*n_1} * \ldots * F_K^{*n_K})(\mathbf{R})\frac{\rho_1^{n_1}\cdots\rho_K^{n_K}}{n_1!\cdots n_K!} \qquad (7)$$

$*$ – convolution symbol

- Blocking probability of class k customers: $B_k = 1-\frac{G_k}{G}$, (8)

$$G_k = \sum_{n_1+\ldots+n_K< S}(F_1^{*n_1} * \ldots * F_k^{*(n_k+1)} * \ldots * F_K^{*n_K})(\mathbf{R})\frac{\rho_1^{n_1}\cdots\rho_K^{n_K}}{n_1!\cdots n_K!}$$

Loss systems with dependent resource requirements and service time

- Acquisition of multiple resources of different types:
  - There are R_m units of resources of type m, $\mathbf{R} = (R_1, \dots, R_M)$
  - The i-th customer of class k requires to hold $r_{km}(i)$ units of resources of type m.
- Service times $\beta_k(i)$ and resource demands $\mathbf{r}_k(i)$, of class k customers, i =1,2,…, have joint cumulative distribution functions

$$H_k(t, \mathbf{x}) = P\{\beta_k(j) \le t, \mathbf{r}_k(j) \le \mathbf{x}\}$$

- Stationary distribution $P_{\mathbf{n}}(\mathbf{x}_1, \dots, \mathbf{x}_K)$ of the system is exactly the same as for the system, in whch service times and resource demands are independent, service times are exponentially distributed with the rate $\mu_k = 1/b_k$ and probability distribution functions of resource requirements $F_k(\mathbf{x})$ given by

$$b_k = \lim_{\substack{x_1 \to \infty \\ \dots \\ x_K \to \infty}} \int_0^\infty t H_k(dt, \mathbf{x}) \qquad F_k(\mathbf{x}) = \frac{1}{b_k} \int_0^\infty t H_k(dt, \mathbf{x})$$

Loss systems with random resource demands

- Acquisition of multiple *resources of different types:*
  - There are R_m units of resources of type m, $\mathbf{R} = (R_1, \ldots, R_M)$
  - The i-th customer of class k requires to hold $r_{km}(i)$ units of resources of type m.
  - Resource demands $\mathbf{r}_k(i) = (r_{k1}(i), \ldots, r_{kM}(i))$, $i = 1, 2, \ldots$ of class k customers are nonnegative random vectors with cumulative distribution functions $F_k(\mathbf{x})$
- The set of feasible states is given by

$$\mathcal{X} = \{(\mathbf{n}, \boldsymbol{\gamma}_1, \ldots, \boldsymbol{\gamma}_K) \mid \mathbf{n} \in \mathbb{N}^K, \boldsymbol{\gamma}_k \in \mathbb{R}_+^M, k = 1, 2, \ldots, K,$$
$$\sum_{k=1}^{K} \boldsymbol{\gamma}_k \le \mathbf{R}, \sum_{k=1}^{K} n_k \le S\}$$

$\mathbf{n} = (n_1, \ldots, n_K)$ – population vector

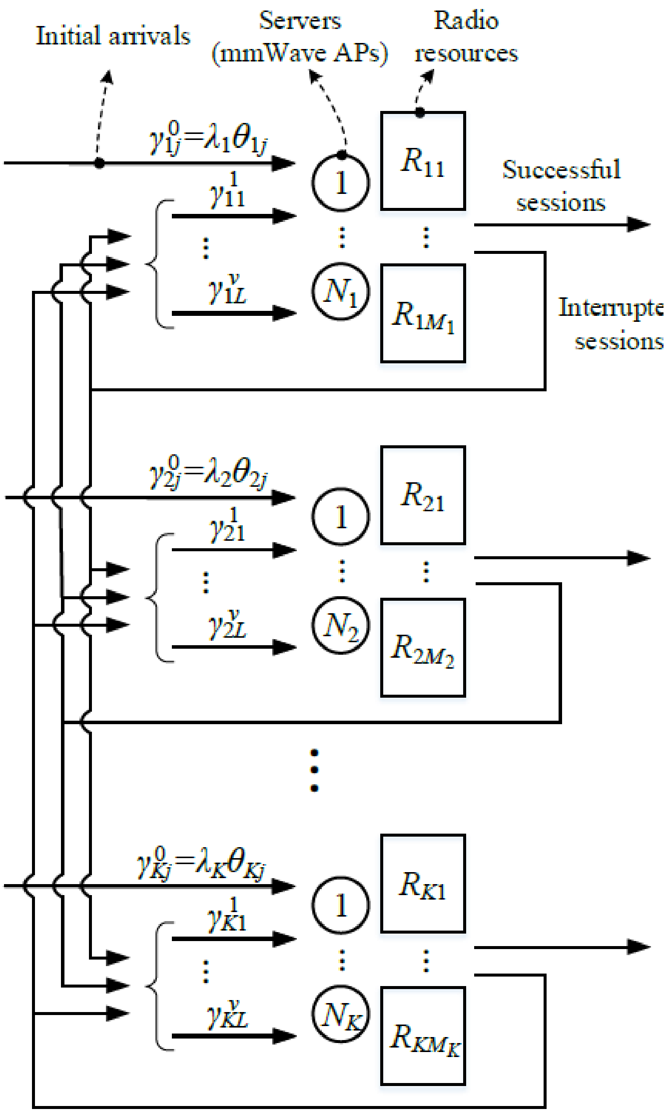
$\boldsymbol{\gamma}_k = (\gamma_{k1}, \ldots, \gamma_{kM})$ – vector of resources occupied by class k customers

Loss systems with positive and negative resource demands

- Resource demands $\mathbf{r}_k(i) = (r_{k1}(i), \ldots, r_{kM}(i)),\;\; i = 1, 2, \ldots$ of class k customers are random ~~nonnegative~~ vectors with cumulative distribution function $F_k(\mathbf{x})$
- Acquisition of a *positive* quantity of a resource means *subtraction* of this quantity from the pool of available resources
- Acquisition of a *negative* quantity of a resource means *addition* of this quantity to the pool of available resources
- A customer with negative resource demand can leave the system only if the resource that was added to the pool of available resources can be picked up without disrupting the service of other calls.

Loss networks with random resource demands and signals

- Network contains customers and signals
- Arriving signal interrupts the service of a customer and forces a customer to leave the network, or to move instantaneously to another loss system where the customer requests new service.
- If the service of a customer was not interrupted, the customer leaves the network and is considered as successfully served.



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